A fundamental uncertainty of spectrometric stationarity

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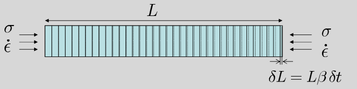
Slow instrument drifts

Clocks – slow acceleration ~ period τ
Spectrometers – slow drift of frequency graduations $\hat{k}$

normalized drift rate $\beta = -\hat{k}^{-1}\, d\hat{k}/dt = -\tau^{-1}\, d\tau/dt$



$$\delta L = L\beta\,\delta t$$

Gravitational compressive stress & creep

There is a compressive stress on & near the earth due to curvature of earth's gravitational field.

General empirical creep formula:

$$\dot{\epsilon} \approx \frac{d\epsilon}{dt} = C d^{-m} \sigma^n e^{-W_d/k_B T}$$

For small and relatively steady stresses, the creep mechanisms will not vary, so variation due to *d*, *m* can be ignored, reducing the residual creep formula to:

$$\dot{\epsilon} \approx C_1 \sigma^n e^{-W_d/k_B T}$$

In most solids, the dislocation barrier $W_d \approx 1\text{-}2$ eV hence

$$p_d = e^{-W_d/k_B T} \approx 10^{-11}\text{-}10^{-21}$$

This is not zero – so a residual creep is unavoidable.

The residual creep would be amplified by tides and is astronomically significant, as explained ahead.

Fundamental instrumentation uncertainty

By definition of β (left) where T_E denotes an *empirical lower bound on instrument rigidity*.

$$\Delta\beta\,\Delta T_E \approx 1$$

Conservative empirical bounds

Age of the earth or the solar system is a conservative *upper bound on rigidity*

$$\Delta T_E \ll T_\odot \approx 4.9 \text{ Ga} \approx 1.55 \times 10^{17} \text{ s}$$

Yields a *lower bound on* β :

$$\Delta\beta \gg (\Delta T_E)^{-1} \approx 6.5 \times 10^{-18} \text{ s}^{-1}$$

This could be significant since

$$H_0 \approx 71 \text{ km s}^{-1}\text{Mpc}^{-1} \approx 2.3 \times 10^{-18} \text{ s}^{-1} \ll \Delta\beta$$

Cosmological significance

Shown right is that β will cause shifts resembling Hubble's law

$$\delta z(\omega) \equiv \delta\omega/\omega = \beta r/c$$

As the cause is local, the shifts should be merely apparent, and cannot suffer gravitational deceleration. But this means acceleration, since a source at distance *r* must now recede twice as fast when it gets to *2r*. In general,

$$\dot{v}(r) = \dot{r}\alpha + r\dot{\alpha} = r\alpha^2 + r\dot{\alpha} = r(\alpha^2 + \dot{\alpha}) \quad \text{where } \alpha \equiv \beta/c$$

yielding a *deceleration coefficient* $q \equiv (-1 + \dot{\alpha}/\alpha^2) = -1$

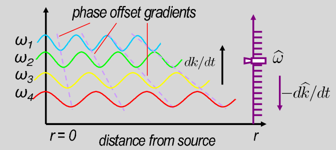
This *exactly* matches the cosmological acceleration, for which

$$q = -1 \pm 0.4$$

[Riess *et al*, 1998]

and resolves several short-range issues.

The spectral scale of space



Fourier theory does not account for change of *static phase offsets* due to change of instrument scale.

As the instrument's graduation marks $\hat{\omega}$ drift, relative to the world, it *requires* a corresponding change in the source path delay to maintain the same phase offsets.

Conversely, *a static source path delay* t = r / c *should appear to the instrument as a changing phase offset, i.e. a frequency shift* $\delta\omega$

World View Source distance *r* not changed, but instrument graduation $\hat{\omega}$ drifts at rate $-d\hat{k}/dt$. Instrument receives successively different frequencies ω_1, ω_2, ω_3, … at the same (drifting) graduation mark $\hat{\omega}$. These frequencies arrive with phase offsets that change successively as

$$\Delta\phi_{ij} \equiv (\omega_j - \omega_i) r/c$$

As the offsets change continuously, they yield the frequency shift

$$\delta\omega \equiv \frac{d\phi}{dt} = \lim_{\Delta t_{ij}\to 0} \frac{\Delta\phi_{ij}}{\Delta t_{ij}} = \lim_{\Delta t_{ij}\to 0} \frac{(\omega_j - \omega_i) r/c}{\Delta t_{ij}} = \lim_{\Delta t_{ij}\to 0} \frac{\Delta k_{ij} r}{\Delta t_{ij}} = r\frac{dk}{dt}$$

with the normalized shift (*scaling*) factor

$$\delta z \equiv \frac{\delta\omega}{\hat{\omega}} = \frac{r}{\hat{k}c}\frac{dk}{dt} = -\frac{r}{\hat{k}c}\frac{d\hat{k}}{dt} = \beta r/c$$

Instrument View Each pure tone in an arriving signal poses the changing offsets, in reverse direction to the world view. The analysis is similar.

The complete frequency shift derivation

Traditional Fourier states and orthogonality given by

$$\langle\hat{\omega}|\omega, r\rangle = \int e^{-i\hat{\omega}t} e^{-i(kr-\omega t)}\, dt = e^{-ikr}\delta(\hat{\omega} - \omega)$$

but this is incomplete, as just noted. From the full phase expression for a sinusoid

$$\phi = \omega(t - r/c) \equiv -(kr - \omega t)$$

the total derivative is

$$\frac{d\phi}{dt} \equiv \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial r}\frac{dr}{dt} + \frac{\partial\phi}{\partial k}\frac{dk}{dt}$$

The first term evaluates to the intrinsic frequency $\frac{\partial[-(kr-\omega t)]}{\partial k} \equiv \partial(\omega t)/\partial t = \omega$

The second term is the familiar Doppler shift since *v = dr/dt* represents recessive velocity. (It is correct but for the Lorentz correction for *v*.) $\frac{\partial[-(kr-\omega t)]}{\partial k}\dot{r} = -k\dot{r} \equiv -\omega v/c$

The third term accounts for instrument scale drift and yields the frequency shift anticipated above. $\frac{\partial[-(kr-\omega t)]}{\partial k}\dot{k} = -r\dot{k} \equiv k\beta r = \omega\beta r/c$

The measured frequency is thus $\omega' \equiv \frac{d\phi}{dt} = \omega(1 - v/c + \beta r/c)$

This is indeed most general, since we finally exhaust all variables in the phase **and** *their interaction on both sides of the dot product.*

"Drift-aware" basis

Traditional Fourier states and orthogonality given by

$$\langle\hat{\omega}|\omega, r\rangle = \int e^{-i\hat{\omega}t} e^{-i(kr-\omega t)}\, dt = e^{-ikr}\delta(\hat{\omega} - \omega)$$

To correct for drift, using the preceding arguments, we should have the modified orthogonality condition

$$\begin{aligned}
\langle\hat{\omega}, \beta|\omega, r\rangle &\equiv \int e^{-i\hat{\omega}t} e^{-i(d\phi/dt)(t-r/c)}\, dt \\
&= \int e^{-i\hat{\omega}t} e^{i[\phi_{,t} + \phi_{,r}\dot{r} + \phi_{,k}\dot{k}](t-r/c)}\, dt \\
&= \int e^{-i\hat{\omega}t} e^{i\omega(1-v/c+\beta r/c)(t-r/c)}\, dt \\
&= e^{-i(k\Delta)r}\delta(\hat{\omega} - \omega\Delta)
\end{aligned}$$

where

$$\Delta \equiv (1 - v/c + \beta r/c)$$

is the total frequency scale factor including Doppler shift if any.

"Drift-aware" analysis

The drift-modified spectra can be related to the original source spectra using the inverse transform as follows.

Starting with the traditional transform pair

$$F(\omega) \equiv \langle\omega|f\rangle = \int \langle\omega|t\rangle\, dt\, \langle t|f\rangle = \int e^{-i\omega t} f(t)\, dt$$

$$\langle t|f\rangle = \int \langle t|\omega\rangle\, d\omega\, \langle\omega|f\rangle = \int e^{i\omega t} F(\omega)\, d\omega$$

first extend it to correct for the path delay, obtaining

$$\begin{aligned}
F(\omega, r) \equiv \langle\omega|f, r\rangle &= \int e^{-i\omega t} f(t - r/c)\, dt \\
&= \int e^{-i(kr+\omega t)} f(t)\, dt = e^{-ikr} F(\omega)
\end{aligned}$$

and

$$\langle t|f, r\rangle = \int \langle t|\omega\rangle\, d\omega\, \langle\omega|f, r\rangle = e^{-ikr}\langle t|f\rangle$$

respectively.
The drift-aware amplitudes are then

$$\begin{aligned}
\langle\hat{\omega}, \beta|f, r\rangle &\equiv \int_t \langle\hat{\omega}, \beta|t\rangle\, dt\, \langle t|f, r\rangle \\
&= \int_t e^{-i\hat{\omega}t}\, dt \int_\omega e^{-i(d\phi/dt)(t-r/c)} F(\omega)\, d\omega \\
&= \int_\omega e^{-i\hat{k}r}\delta(\hat{\omega} - \omega\Delta) F(\omega)\, d\omega \\
&= e^{-i\hat{k}r} F(\hat{\omega}/\Delta)
\end{aligned}$$

proving the drift-induced shift for an arbitrary spectrum.

Properties of drift-aware spectral analysis

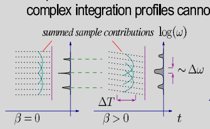
Exact Fourier decomposition is an unverifiable special case.

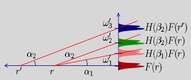
Fourier analysis corresponds to *β = 0* exactly, and is therefore a special case. This was the only case in which the static phase offsets didn't matter (figure below left).

It is unverifiable because the only way to verify the *absence* of drift is to observe sources at *infinity*. The big bang model sets a bound of $r = c / H_0$ on such verification. Even otherwise, the r^2 attenuation of space would make it impossible to observe even galactic sources beyond a finite range.

The drift-aware analysis is most general.

The drift-aware amplitude analysis identifies integration along an inclined profile in the time-frequency domain, shown left. It corresponds to the left factor in the dot product and suffices to exploit the phase offsets. This exhausts the physical information of space that was overlooked in Fourier analysis, so more complex integration profiles cannot do better.





The drift-aware analysis permits instant, monostatic triangulation..

α provides frequency-domain parallax, complementary to spatial parallax given by change of viewing angle (figure above right). (It comes from the right factor in the dot product.)

In particular, we may define an *H* operator by $\langle\hat{\omega}, \beta| \equiv \langle\hat{\omega}|H(\beta)$

It is then straightforward to verify the linearity

$$H(a\beta_1)H(b\beta_2) = H(a\beta_1 + b\beta_2) = H(b\beta_2)H(a\beta_1)$$

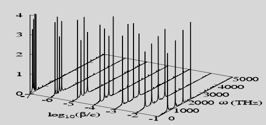
the identity & inverse relations: $H(0) = 1 \quad H(-\beta) = H^{-1}(\beta)$

Separation of signals by source distance is then elegantly given by the project operators $H^{-1}(\beta)GH(\beta)$ where the *G*'s denote band-pass filters.

The drift-aware analysis is causal.

Causality lies in the independence of *β* acting at the instrument from the specific source ranges *r*, *r'*, … In particular, the same *H*(*β*) operator will cause their signals to separate in the frequency domain, as illustrated in the figure for $\alpha_2 \equiv \beta_2/c$.

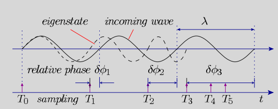
Digital tests with simulated sources



Graph shows linear separations obtained for 4 100 THz simulated sources assumed to be 10^8 m (100,000 km), $2\text{x}10^8$ m, $3\text{x}10^8$ m and $4\text{x}10^8$ m, using an open source Java FFT implementation and a *drifting sampling clock*. Equivalence of the drifting clock to spectrometer drift follows from the phase offset gradient relation

$$\frac{\partial\phi}{\partial\tau}\frac{d\tau}{dt} = \frac{-2\pi}{Nc\tau}\frac{\partial\phi}{\partial k}\cdot\frac{1}{\tau}\frac{d\tau}{dt} = \frac{\omega_\tau}{c}\frac{\partial\phi}{\partial k}\cdot\frac{1}{k}\frac{dk}{dt} = \frac{\partial\phi}{\partial k}\frac{dk}{dt}$$

where ω_τ denotes the sampling frequency, and *N* is the FFT frame size. Figure below illustrates this relation.



The source distances and frequency were chosen to be as close to astronomical distances and visible light as currently possible with the 32-bit trigonometry libraries. The overall logarithmic behaviour of the effective *α* is not yet fully understood, and truncation errors in the math library are also suspect.

Other tests needed

RF tests To test the sufficiency for Continuous Wave source separation and for potential application to communication & imaging.

With gratings To test the possibility of actual transformation of wavelengths, since the integration combines different frequencies with phase offsets exactly as if compressing or expanding in time.

Implications for general physics & metrology

General assumption of stationary states is challenged.

The implications of this remain to be investigated.

Inadequacy of Allan variances for validating atomic clocks

Allan variances are derived from the same clock or a PLL-synchronized second clock in the same vicinity, which would be likely subject to the same gravitational and tidal stresses, and therefore accelerating the same way. For example, NASA's tests for the equivalent clock slow-down in the Pioneer 10/11 anomaly merely verified this local consistency of (potentially similarly accelerating) clocks. The calibration-propagation issue (below) adds to this problem.

Sample issues posed for astronomy

Telescope calibration procedures inadvertently propagate H_0

Calibration is invariably made for both offset and gradient errors by reference to known nebulae *j* at distances r_j say of prior values

$$\omega_{0i} = \omega_i(1 + r_j h_0)$$

where h_0 is the reference value for H_0. The same lines may appear, in the test instrument, at graduations corresponding to say

$$\omega_{si} = \omega_i(1 + r_j h_s + \epsilon)$$

where *ε* denotes offset error and h_s gradient error. Calibration means determining compensations *ε'* and h'_s, respectively, such that we have a good (least squares) fit over the full set of referents, i.e.

$$\omega_i = \frac{\omega_{0i}}{1 + r_j h_0} = \frac{\omega_{si}}{1 + r_j(h_s + h'_s) + (\epsilon - \epsilon')}$$

However, this not only eliminates the offset error, but also ensures $h_s + h'_s = h_0$ so that *all subsequent observations will fit* h_0 *regardless of the truth*. Note that this was not a relevant problem till 1998 because the expansion was thought to be decelerating and hence nonlinear. Reconsideration is now necessary.

Sign and magnitude match for H_0 *and* Λ *from instrument creep*

The compressive stress, combined with kneading by the tides, is of the right sign to cause a geologically slow shrinkage of clocks and spectrometers, to thereby cause old light to appear redshifted, and to dilate past time intervals in the same proportion to age or distance. As remarked above, the *uncertainty* alone exceeds the Hubble constant, *and* the apparent recessions that this would cause would exhibit *exactly* the same deceleration coefficient as the observed Λ. It also gives an *exact* fit for tidal friction and would resolve other short range problems.

Other standard model features have been at least cursorily examined and appear to be equally vulnerable, including, notably, Olbers' paradox and the cosmic microwave background. In any case, this is a *mundane, local cause mandated by solid state physics* under the compressive tidal stress conditions on and near the earth, and needs to be corrected for in all of current physics.

Further information

Application to gratings and DSP was treated in IEEE WCNC'2005 and MILCOM'2005 conference papers. The full simulation is online as an applet at http://www.inspiredresearch.com with detailed discussion. Comments are respectfully solicited.

Updated to reflect comments by Prof. D. Davis of Orange Count CC, NY, hereby gratefully acknowledged.