Planck's law proves radiation is not inherently quantized

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Planck's law itself, a little used 1905 result by Einstein, and the equivalence of frequency transitions to time dilations in finite wave trains, prove (a) that the form of the blackbody spectrum is simply the Doppler spread of individual wave trains by reflection from cavity walls vibrating at the equilibrium temperature; and (b) that wave train energies are quantized only by the discreteness and constancy of the structure of the emitting atoms. Classical equipartition, between wave trains and wall vibrations has thus been responsible for the equilibrium spectrum all along. The participation of wall thermal vibrations, never previously considered, further resolves Loschmidt's paradox in the kinetic theory.

A fundamental distinction of spectra over the composite waveforms they represent is that spectral components are computational artefacts, and being invariably defined for $\pm\infty$, they can only appear or vanish in their entirety in any observation. The amplification or decay of even a classical wave spectral components can only occur end-to-end, since the amplitude of a Fourier component cannot vary with the coordinates. Non-locality is thus a fundamental property of spectral states, rather than an inherently quantum result as hitherto believed, as proved by identical reproduction of a well known example of the EPR paradox by classical electrical field components in the appendix; classical fields work identically because quantization concerns amplitude and not extent, as in digital signal processing. A machine like Deep Blue, driven solely by observational data and not preceding human thought, would not inherit this confusion.

A further emergent problem for current ideas is that the constancy of component (angular) frequencies ω is also not intrinsic, so the energy quanta $\hbar\omega$ given specifically for components in second quantization cannot be fundamental. Chirp decompositions that yield spectral components with frequencies as functions of time or space [1–4] are merely harder to achieve at optical wavelengths [5]. Chirp telemetry spectra, with range proportional shifts and excess delays irreducible to sinusoidal wave properties, have been revealed in the flyby anomaly by undelayed radar data [6].

Quantization then cannot be intrinsic to the state space formalism, since all state functions are by definition spectral components, and thus inherently computational artefacts, nor, by that token, to standing waves and modes, as the latter constitute a spectral representation of the equilibrium state in the blackbody spectrum theory. Use of standing waves and modes as dynamical entities in analogy to gas molecules in the kinetic theory not only leads to a divergent equipartition law, but obfuscates the very dynamics of radiation equilibration, since waves cannot dynamically interact like molecules and their interactions with matter are limited to Doppler spreads around atomic resonances. Even the scattering of wavefronts by reflection at the cavity walls is inadequate, because the resulting random distribution of peaks and troughs [7] must further behave as Planck's oscillators, wherein each transition of frequency by a factor n also increases the energy by exactly n and decreases the probability to an exactly corresponding amount.

Shown here is that a rarely used 1905 result of Einstein is necessary and sufficient to cause exactly this behaviour in the wave trains emitted and absorbed in atomic state transitions, in combination with the Doppler shifts from cavity walls also vibrating at the equilibrium temperature. The wall vibrations are required for the premise of equilibrium, since by the third law [8], non-vibrating walls must be at absolute zero temperature, but have escaped consideration in part because Wien's law considers wall Doppler shifts only for adiabatic macroscopic motions [7]. The wall vibrations provide spectral dispersal by Doppler shifts and the blackbody spectral profile then emerges as the spectrum of each individual wave train until its absorption, involving no interaction between wave trains, hence with no implication of an expectation value for their energies. An expectation value arises only in detailed balance with atomic transitions, requiring *wave train* energies of $\hbar\omega$. The radiation spectrum is thus dictated in form by a purely classical equilibrium of wave trains and wall vibrations, and quantized by atomic interactions not involved in spectral equilibration.

Einstein's notion of "light complexes" with the Doppler energy increases [9] suggests a dependence on his quantum heuristic [10–13], but the increases come independently from the proportional increase of amplitude with frequency in the Lorentz transform, as the quadratic rise in energy density and linear decrease of wave periods lead to a net linear increase of total energy in finite wave trains [14]. The only mathematical justification for the notion of complexes is that a *finite* wave train must have an *infinite*, continuous Fourier spectrum [5] and is a wave packet in that sense.

I. DOPPLER EFFECTS ON WAVE ENERGY AND TIME

The frequency and energy increase by the same factor in the Doppler effect, as discovered by Einstein [9], since

$$E' = E\sqrt{\frac{1-v/c}{1+v/c}} \quad \text{along with} \quad \nu' = \nu\sqrt{\frac{1-v/c}{1+v/c}}. \tag{1}$$

This anticipates the proportionality of energy to frequency in Planck's law, but has been hitherto seen only as implying motion of a source or a target, rather than as a transformation of waves themselves.

The change of wave periods with Doppler shifts, due to the constancy of the wave speed c, implies a corresponding lengthening or shortening of the wave trains, depending on whether the shifts reduced or increased their frequencies. This is equivalent to scaling of the time frames of arbitrary waveforms $f(t)$ by the Fourier inverse transform as

$$\int_\Omega F(\omega[1+z])\, e^{i\omega t}\, d\omega = \frac{1}{1+z}\, f\left(\frac{t}{1+z}\right), \tag{2}$$

where $z \equiv \Delta\omega/\omega$ denotes a uniform fractional shift of the angular frequencies ω in the Fourier spectrum $F(\omega)$. The scaling of time frames was explictly used by Einstein for deriving eq. (1), and the change in wave periods is used in the alternative derivation by Lorentz transform [14]. Yet, this scaling of time with Doppler shifts is generally overlooked as the Doppler effect is ordinarily explained for sinusoids [15, 16], which extend to $\pm\infty$, and thus make it meaningless to consider overall dilations or compressions. It is irrelevant for standing waves for the same reason.

As a result, the time dilations are distinguishable only in temporal phenomena like supernovae light curves [17, 18], where the shifts and dilations resemble Doppler; particle lifetimes [19] and clock rates [20], as described by the special and general theories of relativity, respectively; and in signal processing, where too it has been occasionally overlooked [21, 22]. In the present context, the time dilations provide the required decrease of probabilities, as follows.

II. EQUILIBRATION BY WALL VIBRATIONS

Each wave train travelling in any direction would be itself further split, and its fragments randomly dispersed, by the wall vibrations. The multiplicative law $p(u) \equiv e^{-u/k_B T} = \exp(-\sum_i u_i/k_B T) \equiv \prod_i p(u_i)$, where u_i are the fragment probabilities, should hold as the fragments would be also offset randomly in phase, and therefore act independently. The Boltzmann distribution is implied for wall vibrations by classical equipartition, and should carry over to the wave train fragments. In absence of real interactions between the wave trains, however, equipartition and detailed balance can make sense only with atomic emissions and absorptions. Nevertheless, every fragment of a wave train must itself exhibit the same spectral distribution as those of any other wave train, and thereby the shape of the Planck spectrum, in equilibrium. This condition is fulfilled by the two Doppler effects given by eqs. (1) and (2), as follows.

If a fragment initially has energy u at angular frequency ω for a time interval τ, its subsequent manifestation at any other angular frequency $\omega' \neq \omega$ would present $u' = u\,\omega'/\omega$ to the total energy at ω', because of the Lorentz increase, assuming the fragment remained intact and lost no energy to matter by absorption. The fragment would also shorten to $\tau' = \tau\,\omega/\omega'$, so its Boltzmann probability $e^{-u'/k_B T}$ at ω' would hold only for the interval τ'.

Its contribution at $\omega' = n\omega$, for a natural number n, is then $nu\, e^{-nu/k_B T}$, representing n contiguous appearances at ω' to cover its original interval τ. The energy factor n cannot be simply due to the repeated appearances, however, because the energy integral of a wave depends on integration time and the amplitude, and not on the number of cycles involved. The energy increase must indeed come from eqs. (1), implying energy absorbed from wall vibrations with the Doppler shift, together with a corresponding quantity of momentum, just as in interactions with molecules treated in Einstein's 1917 paper predicting stimulated emission [23]. Equipartition should hold between the wall vibrations and the wave trains because the energy increases in upward Doppler shifts would be balanced by decreases from downward Doppler shifts in equilibrium. This implication of $k_B T/2$ energy expectation for wave trains is fundamentally different from Rayleigh's theory, because the number of wave trains remain finite, whereas the number of standing wave *modes* is *a priori* constrained by geometry as infinite. The historical divergence in Rayleigh's theory was thus due to not so much the wrong choice for the physical entities, as inattention to the mathematical properties of that choice.

The sufficiency of classical mechanics for predicting the blackbody spectrum follows from considering the detailed balance between the fragment's contributions at ω and ω', in the equilibrium steady state, which requires

$$(n+1)u\, e^{-(n+1)u/k_B T} = nu\, e^{-nu/k_B T}, \tag{3}$$

as the energy flow rates would be proportional to probable energy densities. This balance condition implies

$$u(n\omega) = nu(\omega) \quad \text{and} \quad u(n\omega) = \frac{u}{e^{nu/k_B T} - 1} \tag{4}$$

for the time averaged contributions at ω and its harmonics by the initial wave train fragment. The first of eqs. (4) resembles Planck's quantization rule $E = \hbar\omega$, but cannot use $\hbar$ as its scale factor as the balance is between a fragment's own contributions, and not between wave trains. The second equation reproduces the shape of blackbody spectrum, including the cut-off at high frequency, thought classically impossible because of the errors in Rayleigh's theory.

These arguments also explain Planck's harmonic oscillators in the equilibrium picture. The time average over the initial interval τ of a wave train fragment must equal the average of the sum of its contributed energies

$$U = u\, e^{-u/k_B T} + 2u\, e^{-2u/k_B T} + 3u\, e^{-3u/k_B T} + \ldots \tag{5}$$

at ω, 2ω, $\ldots$, over $P = e^{-u/k_B T} + e^{-2u/k_B T} + \ldots$ "states", hence U/P yields the final expression in eq. (4). Planck's oscillators thus describe equilibrium spectra of wave train fragments under Doppler equilibration but for the factor $\hbar$, which is applicable only to the whole wave trains emitted or absorbed by atoms, as treated by Einstein [23].

III. CONCLUSION

Wall thermal vibrations have never been considered, in retrospect, since the gas laws were successfully inferred from the bouncing of gas molecules on stationary walls, and their velocity distribution had followed statistically. Only the abstraction of an entropy principle posed the further difficulty of Loschmidt's paradox [24], which remained unsolved, or what is worse in science, got accepted as unsolvable [25]. Wien's law was a first step in considering wall motions, and yielded h in Planck's and Einstein's derivations thanks to its inclusion of atomic interactions via empirical constants. The present consideration of wall thermal vibrations has been overdue since Nernst's theorem [8], whereby the cavity walls need to be at absolute zero temperature to not vibrate. The equilibrium of any confined volume of radiation or gas at any temperature $T > 0$ is thus really only meaningful if the confining walls are at the same temperature, but the wall vibrations then provide a causal pathway to the rest of the universe, and thus suffice to explain irreversibility in general. The wall vibrations would need to be expressly added to Fermi-Pasta-Ulam models (cf. [26–29]).

Correspondingly, the success of macroscopic thermodynamic and entropy principles in yielding Wien's law and the blackbody spectrum in Planck's and Einstein's derivations thus does not validate its completeness. Instead, the lack of detailed consideration of the radiation equilibration process, treated for the first time here, also limits other recent suggestions of a classical origin of Planck's law [30–34] and of zero-point energy [32, 35, 36].

The intrinsic non-locality of wave spectral components anticipates the other exotic properties of quantum mechanics, The creation and annihilation operators in second quantization are also not inherently quantum: any strengthening or weakening of a classical wave component must similarly change its amplitude in parallel for all time $t \in (-\infty, +\infty)$ and simultaneously at all distances $r \in (-\infty, +\infty)$, as remarked. The operators work in quantum theory only because spectral components are computational artefacts, to begin with. The inherent non-locality is particularly illustrated by reproducing a known example of the EPR paradox in the Appendix, showing that the current notions of entanglement confuse the domain of spectral components with their amplitudes, as remarked. The only inherent discreteness in nature is of matter itself, and radiation quantization is mostly Einstein's heuristic, for lack of distinction between wave trains and spectra. The spectral decomposition of cyclic states in Hamilton-Jacobi theory was simply not developed prior to Born-Jordan theory (cf. [37–40]), which led to anti-commutators and Poisson brackets that either all vanished, so they could represent no stationary structures at all, or reduced to multiples of h from radiation theory [41, §21], as both concern the structure of matter.

The probabilities represented by particle wavefunctions cannot signify particle thermal fluctuations, but this too is now easily explained and related to classical insight. More so than irreversibility in observational processes [42, 43], a physical observer oneself must have a positive temperature for the change of state to embody the resulting information, again by the third law, so fluctuations occur in observations, and make the observer's state transitions probabilistic.

Appendix A: Identical non-locality of classical wave spectral components

Feynman's example for the EPR paradox involves circularly polarized gamma photons from a decaying positronium source in $j = 0$ state [16, III-18-3]. The linear polarizations of the photons are then detected by analyzers located in opposite directions from the source, so any correlation between the analyzer readings seems unpredictable without precise knowledge of their locations to within a gamma wavelength. This classical unpredictability is thought especially ensured in Aspect's "time-of-flight tests" by switching the analyzer orientations after photon pair production [45–47].

In the quantum description, conservation of the angular momentum $j = 0$ involves emitted photon states $|l_1 l_2\rangle \equiv |l_1\rangle|l_2\rangle$ or $|r_1 r_2\rangle \equiv |r_1\rangle|r_2\rangle$, i.e., both left or right circularly polarized, using letters for polarization and subscripts for the particles, respectively. Assuming odd parity, the final state is $|\psi\rangle = |l_1 l_2\rangle - |r_1 r_2\rangle$. The possible combinations of the analyzer states are $|x_1 x_2\rangle, |x_1 y_2\rangle, |y_1 x_2\rangle$ or $|y_1 y_2\rangle$, where x and y denote Cartesian axes, with the amplitudes

$$\begin{aligned} \langle x_1 x_2|\psi\rangle &= \langle x_1|l_1\rangle\langle x_2|l_2\rangle - \langle x_1|r_1\rangle\langle x_2|r_2\rangle = 0, \\ \langle x_1 y_2|\psi\rangle &= \langle x_1|l_1\rangle\langle y_2|l_2\rangle - \langle x_1|r_1\rangle\langle y_2|r_2\rangle = -i, \end{aligned} \tag{A1}$$

and also $\langle y_1 y_2|\psi\rangle \equiv \langle x_1 x_2|\psi\rangle$ and $\langle x_2 y_1|\psi\rangle \equiv -\langle x_1 y_2|\psi\rangle$, since $\langle x_1|l_1\rangle = \langle x_2|l_2\rangle = \langle x_1|r_1\rangle = \langle x_2|r_2\rangle = 1/\sqrt{2}$, and $\langle y_2|l_2\rangle = -i/\sqrt{2} = -\langle y_2|r_2\rangle$. Non-zero amplitudes imply the observations would be correlated, though the analyzers are not coupled and the photons travel independently.

The corresponding classical circularly polarized Fourier spectral components of the electric field, with wave vector k and initial phase offset α, would be

$$|x\rangle \sim \mathbf{e}_x E_x e^{i(\alpha+kz-\omega t)} \quad \text{and} \quad |y\rangle \sim \mathbf{e}_y E_y e^{i(\alpha+kz-\omega t+\pi/2)} = i\mathbf{e}_y E_y e^{i(\alpha+kz-\omega t)}, \tag{A2}$$

where the quantum states are shown at left for clarity; E_x and E_y are the transverse electric field amplitudes; and z is the direction of propagation. The phase offset α remains constant and can be safely ignored.

Relative phase offsets $\pm\pi/2$ in y and amplitude relation $E_x = E_y = E$ define the circular polarizations as

$$\begin{aligned} \sqrt{2}|r\rangle = |x\rangle + |y\rangle \sim &\quad (\mathbf{e}_x E_x + i\mathbf{e}_y E_y)\ e^{i(kz-\omega t)} \\ \text{and} \quad \sqrt{2}|l\rangle = |x\rangle - |y\rangle \sim &\quad (\mathbf{e}_x E_x - i\mathbf{e}_y E_y)\ e^{i(kz-\omega t)}. \end{aligned} \tag{A3}$$

Classical analyzer states are unit vectors $\langle x| \sim \mathbf{e}_x e^{-i\omega t}$ and $\langle y| \sim \mathbf{e}_y e^{-i\omega t}$, so the detected signal amplitudes are

$$\begin{aligned} \langle x|r\rangle &\sim \int_T \mathbf{e}_x e^{i\omega\tau} \frac{\mathbf{e}_x E_x + i\mathbf{e}_y E_y}{\sqrt{2}}\ e^{i(kz-\omega\tau)} d\tau = \frac{E_x\ e^{ikz}}{\sqrt{2}}, \\ \langle x|l\rangle &\sim \int_T \mathbf{e}_x e^{i\omega\tau} \frac{\mathbf{e}_x E_x - i\mathbf{e}_y E_y}{\sqrt{2}}\ e^{i(kz-\omega\tau)} d\tau = \frac{E_x\ e^{ikz}}{\sqrt{2}}, \\ \langle y|r\rangle &\sim \int_T \mathbf{e}_y e^{i\omega\tau} \frac{\mathbf{e}_x E_x + i\mathbf{e}_y E_y}{\sqrt{2}}\ e^{i(kz-\omega\tau)} d\tau = \frac{iE_y\ e^{ikz}}{\sqrt{2}}, \text{ and} \\ \langle y|l\rangle &\sim \int_T \mathbf{e}_y e^{i\omega\tau} \frac{\mathbf{e}_x E_x - i\mathbf{e}_y E_y}{\sqrt{2}}\ e^{i(kz-\omega\tau)} d\tau = \frac{iE_y\ e^{ikz}}{-\sqrt{2}}, \end{aligned} \tag{A4}$$

where the integrations are for observation times T longer than a wave period, so the result oscillates with distance. As the integrands are Fourier components extending over $t \in (-\infty, +\infty)$, their phases cannot randomly fluctuate during the integration. The product signals in x_1 are then

$$\begin{aligned} \langle x_1 x_2|r_1 r_2\rangle \equiv \langle x_1|r_1\rangle\langle x_2|r_2\rangle &\sim \frac{E_x^2 e^{ik(z_1+z_2)}}{2}, \\ \langle x_1 x_2|l_1 l_2\rangle \equiv \langle x_1|l_1\rangle\langle x_2|l_2\rangle &\sim \frac{E_x^2 e^{ik(z_1+z_2)}}{2}, \\ \langle x_1 y_2|r_1 r_2\rangle \equiv \langle x_1|r_1\rangle\langle y_2|r_2\rangle &\sim i\frac{E_x E_y e^{ik(z_1+z_2)}}{2}, \text{ and} \\ \langle x_1 y_2|l_1 l_2\rangle \equiv \langle x_1|l_1\rangle\langle y_2|l_2\rangle &\sim -i\frac{E_x E_y e^{ik(z_1+z_2)}}{2}. \end{aligned} \tag{A5}$$

A classical entangled state corresponding to $|l_1 l_2\rangle - |r_1 r_2\rangle$ exists since

$$\begin{aligned} \langle x_1 x_2|\psi\rangle \equiv \langle x_1 x_2|l_1 l_2\rangle - \langle x_1 x_2|r_1 r_2\rangle &\sim 0 \text{ identically, and} \\ \langle x_1 y_2|\psi\rangle \equiv \langle x_1 y_2|l_1 l_2\rangle - \langle x_1 y_2|r_1 r_2\rangle &\sim -iE_x E_y e^{ik(z_1+z_2)}, \end{aligned} \tag{A6}$$

i.e., eqs. (A1) with deterministic classical amplitudes. There is ordinarily been no reason to consider such combinations of classical detector signals, especially since physicists perceived the latter as ultimately comprising quanta.

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