Chirp modes for ranging and communication

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Inspired Research, New York

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Concept

- Breakthrough
  - Chirp travelling wave modes, subsume sine waves
  - Overlooked at the start of wave theory $\sim$ 1740s
  - Vindicates d'Alembert over Euler
- General receiver technology, enabling
  - receive any source of wavelength λ_s at any wavelength λ_r
  - unilaterally switch λ_r to overcome jamming or interference
  - receive source wavelength λ_s via channel of wavelength λ_c
  - receive multiple sources $\lambda_1, \lambda_2, \ldots$ via channel wavelength λ_c
  - image at wavelength λ_c by illumination at wavelength λ_s
  - continuously track source range
- Sample impacts
  - Spectrum auctions, frequency agility redundant
  - Wavelength-specific technologies, e.g. THz, redundant
  - Automotive, robot, drone radars with a lot less radiation!
  - Infinite bandwidth on every cell phone, WiFi

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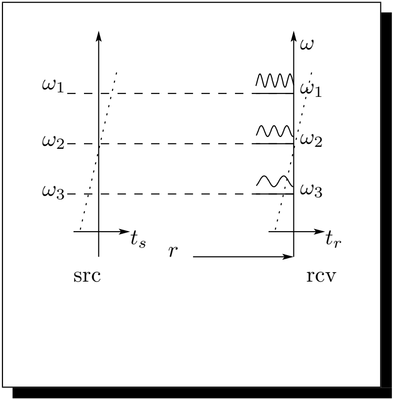
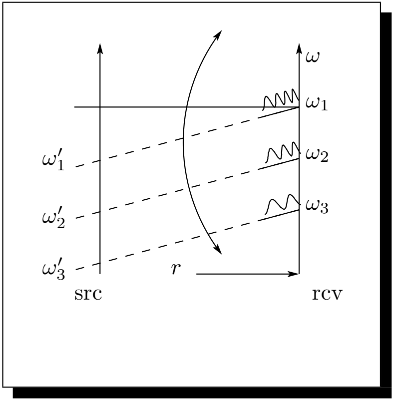
Meaning



Sinusoidal modes (from Euler)
– received at same frequencies
– including chirps (dotted lines)



Chirp modes
– received at offset of choice
– frequency domain parallax
– angle ∼ chirp rate

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Part I

The real general theory of waves

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Vibrating string controversy

- Fourier series really due to Euler and D Bernoulli [1]
  - Fresnel (1805): extension to optics – diffraction
  - Fourier (1807-): inverse, space-time separation, heat theory
  - Dirichlet, Cauchy, Parseval: convergence and completeness
- Fourier inspired by d'Alembert and Euler (both died 1783)
  - d'Alembert solution factors *before* separation
  - Fourier separation *provides* factorization, Euler series
  - *Fourier premise could not have been necessary for d'Alembert*
- Representational completeness is not complete!
  - Fourier means invariance in r AND invariance in t – translational symmetry: $(t, r) \longrightarrow (t - \delta, r - \delta)$
  - d'Alembert calls for invariance in $(t - r/c)$ ONLY – admits scaling symmetry: $(t, r) \longrightarrow (\gamma[t - \delta], \gamma[r - \delta])$
  - *d'Alembert invariance is weaker, more general*
- Above points proved next . . .

[1] I Kleiner, Evolution of the function concept: A brief survey, The College Math J, 20, 4, 282-300 (1989); G F Wheeler and W P Crummett, The vibrating string controversy, Am J Phys, 55, 1, 33-37 (1987)

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Fourier and d'Alembert invariances

- Euler-Bernoulli-Fourier constraint:
  - $f(t, r) = f(t - \delta, r - \delta)$ for arbitrary displacements δ
  - Implies $df/dt = sf$, $df/dr = sr$, where $|s| = 1$
  - Fourier's method:
    - $df/f = s\ dt \Leftrightarrow f = f_0 e^{st}$ ($|s| \neq 1 \rightarrow$ Laplace transforms)
    - As trial solutions, $f_0 e^{st}$ reduce PDE in (t, r) to ODE in r
    - ODE solutions for same $s \Rightarrow$ same displacements δ

- d'Alembert admits expansions and dilations:
  - Theorem: *if $f(t - r)$ is a solution and g is a 2nd order continuous function, $f(g(t - r)[t - r])$ is also a solution.*
  - Proof: $f(g(t - r).[t - r])$ is then also 2nd order continuous in t and r. For any such function $\psi(r, t)$, $\psi_{,t} = \psi'; \psi_{,r} = -\psi' \Leftrightarrow$ $\psi_{,tt} = \psi''; \psi_{,rr} = -\psi'_{,r} = \psi'' \Leftrightarrow \psi_{,tt} - \psi_{,rr} = 0$ $\square$
  - In exponential modes, $g = e^{\beta t}$; formal treatment online

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Chirp modes and spectra

$$f(t - r/c) = \frac{1}{2\pi} \int_{\omega,\beta} \overbrace{F_\beta(\omega)\, \exp(-i\omega e^{\beta[t-r/c]}[t - r/c])}^{\text{chirp spectral component (or mode)}}\, d\omega\, d\beta$$

- Weakest premise for d'Alembert solutions
  - Smooth function in $(t - r/c)$, hence solution
  - Subsumes Fourier, Laplace, z as null subspace: $\beta \in \{0\}$
  - Exponential form simplest: uniform rate parameter β
- Frequencies scale with travel
  - Inverse Fourier $\Rightarrow$ time dilated/compressed waveform
  - Explains how two receivers see different spectra of same source
  - *Scaling reconstructions were historically overlooked* [2]

[2] in the foundation of Fourier theory by Cauchy, Weierstrass, Dirichlet . . . The very notions of non-Cartesian coordinates, enabling us to consider ω non-orthogonal to t, and of time dilations, which explain the two-receiver problem, came much later with Riemann and Einstein, respectively.

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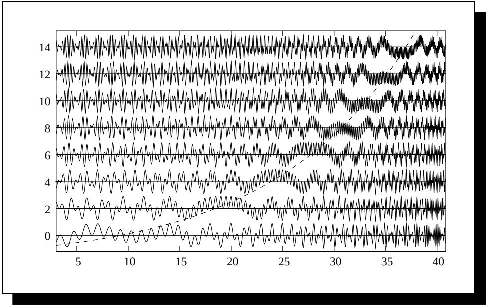
Orthogonal to Fourier modes

Dot products of chirps against a fixed sinusoid



- Each chirp: a small set of cycles contributes at each frequency
  - Infinite spread, zero-measure power
  - Sinusoids *unobservable* by chirped receivers and *vice versa*
- Mutually orthogonal chirp bases distinguished by β
  - Each is a complete Hilbert space!
- **Temporal modes** in receiver's analysis, not about source

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Traditional wave solutions

- Fourier orthogonality

$$\int_T e^{-i\omega(t-r/c)} e^{i\omega_r t}\,dt \equiv e^{i\omega r/c} \int_T e^{i(\omega_r-\omega)t}\,dt \qquad [\omega_r:\ \text{reference}]$$
$$= e^{i\omega\Delta t} \int_T e^{i\omega_e t}\,dt \simeq e^{i\omega_r\Delta t}\delta(\omega_r-\omega)$$

- Fourier decomposition

$$\int_T f(t-r/c) e^{i\omega_r t}\,dt \equiv \int_T e^{i\omega_r t}\,dt \left[\frac{1}{2\pi}\int F(\omega) e^{-i\omega(t-r/c)}\,d\omega\right]$$
$$= \frac{1}{2\pi}\int d\omega \left[F(\omega) e^{i\omega r/c} \int_T e^{i(\omega_r-\omega)t}\,dt\right]$$
$$\simeq \frac{1}{2\pi}\int [F(\omega) e^{i\omega\Delta t}\delta(\omega_r-\omega)]\,d\omega = F(\omega_r) e^{i\omega_r\Delta t}.$$

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Traditional wave solutions

- Fourier orthogonality – kernel is **receiver-applied reference**

$$\int_T e^{-i\omega(t-r/c)} e^{i\omega_r t}\, dt \equiv e^{i\omega r/c} \int_T e^{i(\omega_r-\omega)t}\, dt \qquad [\omega_r :\ \text{reference}]$$

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$$= e^{i\omega\Delta t} \int_T e^{i\omega_e t}\, dt \simeq e^{i\omega_r\Delta t}\delta(\omega_r-\omega)$$

- Fourier decomposition – reference (kernel) ω_r scales phase lag

$$\int_T f(t-r/c) e^{i\omega_r t}\, dt \equiv \int_T e^{i\omega_r t}\, dt \left[\frac{1}{2\pi}\int F(\omega) e^{-i\omega(t-r/c)}\, d\omega\right]$$

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$$= e^{i\omega\Delta t} \int_T e^{i\omega_e t}\, dt \simeq e^{i\omega_r\Delta t}\delta(\omega_r - \omega)$$

- Fourier decomposition – "retardation" $\Delta t \equiv r/c$ scales phase lag

$$\int_T f(t-r/c) e^{i\omega_r t}\, dt \equiv \int_T e^{i\omega_r t}\, dt \left[\frac{1}{2\pi}\int F(\omega) e^{-i\omega(t-r/c)}\, d\omega\right]$$

$$= \frac{1}{2\pi}\int d\omega \left[F(\omega) e^{i\omega r/c} \int_T e^{i(\omega_r-\omega)t}\, dt\right]$$

$$\simeq \frac{1}{2\pi}\int [F(\omega) e^{i\omega\Delta t}\delta(\omega_r - \omega)]\, d\omega = F(\omega_r) e^{i\omega_r\Delta t}.$$

Phase is the cumulative variable, but is cyclic

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Traditional wave solutions

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Phase is the cumulative variable, but is cyclic

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Chirped receiver solutions

- Chirp orthogonality

$$\int_T \exp(-i\omega_0 e^{\beta'[t-r/c]}[t-r/c]) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\approx \exp(i\omega_0\, e^{\beta'[t-r/c]} r/c) \int_T \exp(i\omega_{r0} e^{\beta t} t - i\omega_0 e^{\beta'[t-r/c]} t)\, dt$$
$$\simeq \exp(i\omega_0 e^{\beta'[t-r/c]} r/c)\, \delta(\omega_{r0} - \omega_0 e^{-\beta' r/c})\, \delta(\beta - \beta')$$

- Decomposition

$$\int_T f(t-r/c) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\equiv \int_T \exp(i\omega_{r0} e^{\beta t} t)\, dt \left[\int_{\omega,\beta'} F_{\beta'}(\omega) \exp(-i\omega e^{\beta'[t-r/c]}[t-r/c])\, \frac{d\omega}{2\pi}\, d\beta' \right]$$
$$\simeq \frac{1}{2\pi} \int_{\omega,\beta'} F_{\beta'}(\omega) \exp(i\omega e^{\beta'[t-\Delta t]} \Delta t) \delta(\omega_{r0} - \omega e^{-\beta' \Delta t}) \delta(\beta - \beta')\, d\omega\, d\beta'$$
$$= F_\beta(\omega \exp(i\omega e^{\beta[t-\Delta t]} \Delta t)$$

Chirped receiver solutions

- Chirp orthogonality – apply a reference varying over time

$$\int_T \exp(-i\omega_0 e^{\beta'[t-r/c]}[t-r/c]) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\approx \exp(i\omega_0\, e^{\beta'[t-r/c]} r/c) \int_T \exp(i\omega_{r0} e^{\beta t} t - i\omega_0 e^{\beta'[t-r/c]} t)\, dt$$
$$\simeq \exp(i\omega_0 e^{\beta'[t-r/c]} r/c)\, \delta(\omega_{r0} - \omega_0 e^{-\beta' r/c})\, \delta(\beta - \beta')$$

- Decomposition

$$\int_T f(t - r/c) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\equiv \int_T \exp(i\omega_{r0} e^{\beta t} t)\, dt \left[\int_{\omega,\beta'} F_{\beta'}(\omega) \exp(-i\omega e^{\beta'[t-r/c]}[t - r/c]) \frac{d\omega}{2\pi}\, d\beta' \right]$$
$$\simeq \frac{1}{2\pi} \int_{\omega,\beta'} F_{\beta'}(\omega) \exp(i\omega e^{\beta'[t-\Delta t]} \Delta t) \delta(\omega_{r0} - \omega e^{-\beta' \Delta t}) \delta(\beta - \beta')\, d\omega\, d\beta'$$
$$= F_\beta(\omega \exp(i\omega e^{\beta[t-\Delta t]} \Delta t)$$

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Chirped receiver solutions

- Chirp orthogonality – again incorporating retarded interaction

$$\int_T \exp(-i\omega_0 e^{\beta'[t-r/c]}[t-r/c]) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\approx \exp(i\omega_0\, e^{\beta'[t-r/c]} r/c) \int_T \exp(i\omega_{r0} e^{\beta t} t - i\omega_0 e^{\beta'[t-r/c]} t)\, dt$$
$$\simeq \exp(i\omega_0 e^{\beta'[t-r/c]} r/c)\, \delta(\omega_{r0} - \omega_0 e^{-\beta' r/c})\, \delta(\beta - \beta')$$

- Decomposition

$$\int_T f(t-r/c) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\equiv \int_T \exp(i\omega_{r0} e^{\beta t} t)\, dt \left[\int_{\omega,\beta'} F_{\beta'}(\omega) \exp(-i\omega e^{\beta'[t-r/c]}[t-r/c])\, \frac{d\omega}{2\pi}\, d\beta'\right]$$
$$\simeq \frac{1}{2\pi} \int_{\omega,\beta'} F_{\beta'}(\omega) \exp(i\omega e^{\beta'[t-\Delta t]} \Delta t) \delta(\omega_{r0} - \omega e^{-\beta' \Delta t}) \delta(\beta - \beta')\, d\omega\, d\beta'$$
$$= F_\beta(\omega \exp(i\omega e^{\beta[t-\Delta t]} \Delta t)$$

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Chirped receiver solutions

- Chirp orthogonality – only similar chirps are observable

$$\int_T \exp(-i\omega_0 e^{\beta'[t-r/c]}[t-r/c]) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$

$$\approx \exp(i\omega_0\, e^{\beta'[t-r/c]} r/c) \int_T \exp(i\omega_{r0} e^{\beta t} t - i\omega_0 e^{\beta'[t-r/c]} t)\, dt$$

$$\simeq \exp(i\omega_0 e^{\beta'[t-r/c]} r/c)\, \delta(\omega_{r0} - \omega_0 e^{-\beta' r/c})\, \delta(\beta - \beta')$$

- Decomposition

$$\int_T f(t-r/c) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$

$$\equiv \int_T \exp(i\omega_{r0} e^{\beta t} t)\, dt \left[\int_{\omega,\beta'} F_{\beta'}(\omega) \exp(-i\omega e^{\beta'[t-r/c]}[t-r/c])\, \frac{d\omega}{2\pi}\, d\beta' \right]$$

$$\simeq \frac{1}{2\pi} \int_{\omega,\beta'} F_{\beta'}(\omega) \exp(i\omega e^{\beta'[t-\Delta t]} \Delta t) \delta(\omega_{r0} - \omega e^{-\beta' \Delta t}) \delta(\beta - \beta')\, d\omega\, d\beta'$$

$$= F_\beta(\omega \exp(i\omega e^{\beta[t-\Delta t]} \Delta t)$$

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Chirped receiver solutions

- Chirp orthogonality – frequency ∼ cumulative variable & acyclic

$$\int_T \exp(-i\omega_0 e^{\beta'[t-r/c]}[t-r/c]) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\approx \exp(i\omega_0\, e^{\beta'[t-r/c]} r/c) \int_T \exp(i\omega_{r0} e^{\beta t} t - i\omega_0 e^{\beta'[t-r/c]} t)\, dt$$
$$\simeq \exp(i\omega_0 e^{\beta'[t-r/c]} r/c)\, \delta(\omega_{r0} - \omega_0 e^{-\beta' r/c})\, \delta(\beta - \beta')$$

- Decomposition – lag now scales the frequency or the travel

$$\int_T f(t-r/c) \exp(i\omega_{r0} e^{\beta t} t)\, dt$$
$$\equiv \int_T \exp(i\omega_{r0} e^{\beta t} t)\, dt \left[\int_{\omega,\beta'} F_{\beta'}(\omega) \exp(-i\omega e^{\beta'[t-r/c]}[t-r/c]) \frac{d\omega}{2\pi}\, d\beta'\right]$$
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$$= F_\beta(\omega \exp(i\omega e^{\beta[t-\Delta t]} \Delta t)$$

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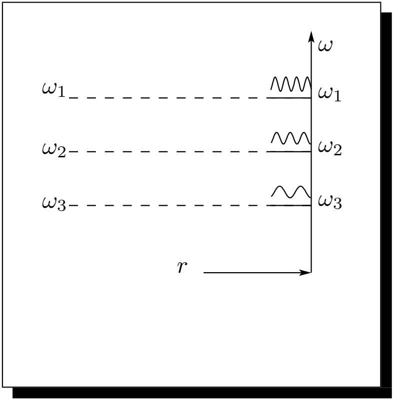
Understanding wave reception



- *Amplitude* $F(\omega_i)$ of each frequency ω_i is obtained by *integrating* the product of the wave with a reference signal $e^{i\omega_i t}$ for short interval T
- *Orthogonality* means 1:1 correspondence with source frequencies ← parallel lines
- Sinusoidal: each amplitude $F(\omega_i)$ belongs to the same frequency ω_i at the source

Duality of existence and computation: using references, we *separate* spectral components, so it's been easy to assume nothing else exists since the Fourier representation was *known* to be "complete".

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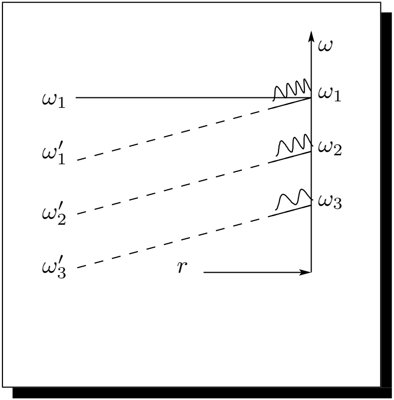
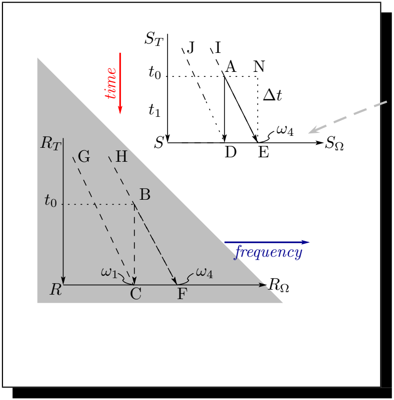
Effect of nonconstant references



- With chirped references $\omega_i(t) = \omega_i(0)e^{\beta t}$, the same integrations at same ω_i yield chirp amplitudes $F(\omega_i(t))$
- Orthogonality to the applied references implies projected source frequencies $\omega_i' \approx \omega_i[1 - \beta r/c]$
- Chirp references $\equiv$ time varying ω scale $\equiv$ clock acceleration
  Not a mere offset of scale

Received *frequencies* ω_i are always same as the applied references. What happens is that the source *amplitudes* at ω_i' appear at ω_i. The Doppler effect of clock acceleration. Parallax in the frequency domain ...

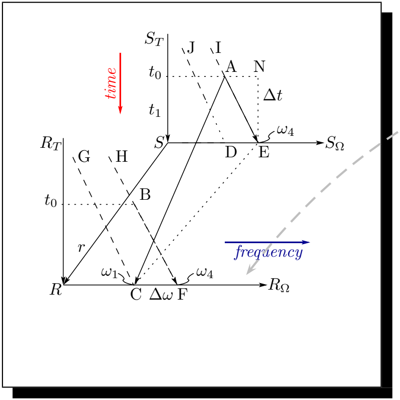
The computed spectral past



- Time axis at source: $S_T S$
  Frequency axis: SS_Ω (at t_2)
  $D \sim$ coefficient for ω_1 at t_2
  $E \sim$ coefficient for ω_4 at t_2
- At t_0 in past: $A \sim \omega_1$, $N \sim \omega_4$
  Fourier – same ω: AD, NE
- Decomposition or selection is never perfect, and may yield lines like AE – A cannot have been coefficient for ω_4 at t_0 – that role belongs to N.

Computed lines do not matter to local history as we cannot go back to t_0 to check if the current coefficient of ω_4 was at A or at N. The lines are merely computational artefacts with respect to local past.

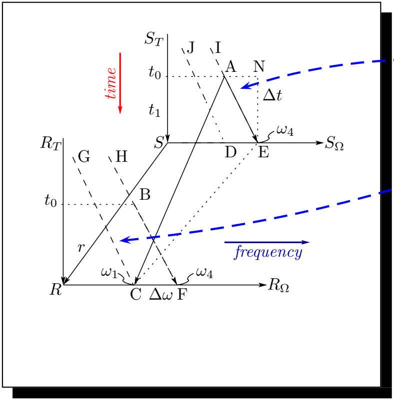
Travel exposes spectral past



- Time axis at receiver: $R_T R$
  Frequency axis: RR_Ω (at t_2)
  $C \sim$ coefficient for ω_1 at t_2
  $F \sim$ coefficient for ω_4 at t_2
- At t_0 in past: $B \sim \omega_1$,
  Fourier – same ω: BC
- Coefficient of A for ω_1 arrives at C, identified with chirp lines $GC \sim \omega_1$ at receiver, $AE \sim \omega_4$ at source, presenting the lag $CF \sim \Delta\omega = \beta\Delta t \equiv \beta r/c$.

Computed lines *define* the received signals or components. Each source coefficient arrives on different receiver component depending on the slant and distance. The shifts violate Fourier, not d'Alembert.

Physics of the exposure



- The integration at C links
  AE: $F_\beta(\omega)\ \exp(-i\omega e^{\beta t'} t')$
  with
  GC: $\exp(i\omega_{r0} e^{\beta t} t)$
  where $t' = t - r/c$
- How did the receiver know the shifts to apply? The wave $f(t)$ arrives "retarded" as $f(t')$
- How can the receiver measure the lag, receiving only ω_1 (C) and not ω_4 (F)?

Local physics yields ω_4 at both source and receiver. A Fourier receiver would report ω_4 at both places. So the lag $\omega_1 - \omega_4$ is always measurable. Difference from phase lags: ω_1 is *precursor* (A) of ω_4 (E) in source history
Delay doubled: $\Delta t(A \to E) + \Delta t(A \to C)$ – caused "flyby anomaly".

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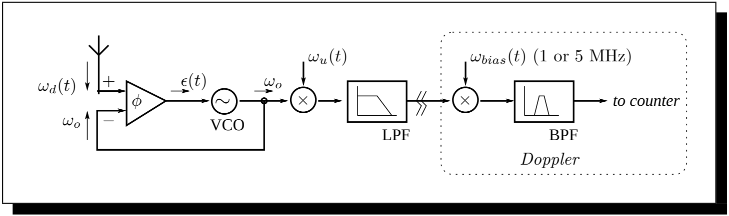
Part II

Implementation

Application to analogue RF reception

DSN Doppler receiver – J D Anderson *et al.*, arXiv:gr-qc/0104064; D K Shin, DSN handbook (2010)



- PLLs involve comparing phases = integrals of frequencies

$$\int_T \underbrace{\exp\left[\frac{i}{\beta'}(\omega_c + \Omega_m)e^{\beta'(t-r/c)}\right]}_{\text{downlink phase}} \underbrace{\exp\left[-\frac{i\omega_o}{\beta}e^{\beta t}\right]}_{\text{VCO phase}} dt$$

$$\simeq 2\pi\,\delta\left(\omega_c + \langle\Omega_m\rangle - \omega_o\right)\delta(\beta' - \beta)$$

ω_d: downlink carrier; Ω_m: instantaneous modulation (PSK); T: time constant of the loop.

- Demodulated signal is $\epsilon(t) \sim \Omega_m$, *from chirp mode if* $\beta \neq 0$
- Implementation rule: inject a ramp into the VCO on top of $\epsilon(t)$ ...

Application to digital receivers

- Standard DFT formula

$$\sum e^{im\omega_\tau(n\tau - r/c)}\ e^{-iln\omega_\tau n\tau} = N\delta_{ml} \sum e^{-im\omega_\tau nr/c}$$

where $\omega_\tau = 2\pi/N\tau$, τ: sampling interval, sums: 0 to $N-1$.

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where $\omega_\tau = 2\pi/N\tau$, τ: sampling interval, sums: 0 to $N-1$.

- With time varying kernel frequencies, this becomes

$$\sum e^{im\omega'_{\tau 0}\beta'^{-1}\exp(\beta'[n\tau - r/c])}\ e^{il\omega_{\tau 0}\beta^{-1}\exp(\beta n\tau)} = N\delta_{ml}\ \delta(\omega'_{\tau 0}e^{\beta'[t-r/c]} - \omega_{\tau 0}e^{\beta t})$$

bearing the frequency lags

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Application to digital receivers

- Standard DFT formula

$$\sum e^{im\omega_\tau(n\tau - r/c)}\ e^{-iln\omega_\tau n\tau} = N\delta_{ml} \sum e^{-im\omega_\tau nr/c}$$

where $\omega_\tau = 2\pi/N\tau$, τ: sampling interval, sums: 0 to $N-1$.

- With time varying kernel frequencies, this becomes

$$\sum e^{im\omega'_{\tau 0}\beta'^{-1}\exp(\beta'[n\tau - r/c])}\ e^{il\omega_{\tau 0}\beta^{-1}\exp(\beta n\tau)}$$
$$= N\delta_{ml}\ \delta(\omega'_{\tau 0}e^{\beta'[t-r/c]} - \omega_{\tau 0}e^{\beta t})$$

bearing the frequency lags

- Options: ramp kernel frequencies ω_τ or the sampling interval τ

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Application to correlation spectroscopy

- Standard autocorrelation and its Fourier transform

$$R(\tau) = \int_T f(t) f^*(t-\tau)\, dt \;\Leftrightarrow\; |F(m\omega)|^2 = \sum e^{im\omega t_n}\, R(t_n)$$

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$$R(\tau) = \int_T f(t) f^*(t-\tau)\, dt \;\Leftrightarrow\; |F(m\omega)|^2 = \sum e^{im\omega t_n}\, R(t_n)$$

- The autocorrelation of a *retarded chirp* is

$$\begin{aligned} A \exp(i\omega\beta^{-1} e^{\beta[t-r/c]}).A^* \exp(-i\omega\beta^{-1} e^{\beta[t-\tau-r/c]}) \\ = A^2 \exp(i\omega\beta^{-1} e^{\beta[t-r/c]}[1-e^{-\beta\tau}]) \end{aligned}$$

  - Frequency lags survive convolution and reach the DFT
  - Power spectra using autocorrelation bear the same shifts

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Application to optical spectroscopy

- Transform kernel is by interference between wavefronts
  - General Snell's law for diffraction and refraction

$$n\dot{\lambda} = (\eta\dot{l} + l\dot{\eta})\sin\theta$$

where n is the order of diffraction; l is the grating interval; and η is the refractive index

  - Let $\dot{l}/l = \gamma\beta$ and $\dot{\eta}/\eta = (1-\gamma)\beta$, where $\gamma \in [0,1)$ then, at each angle θ, wavelength varies at rate $\dot{\lambda}/\lambda = \beta$
  - Interference of wavefronts from $N = L/l$ slits, arriving at successive delays $\tau = l\sin\theta/c$, $\Rightarrow$ same form as DFT

- Varying grating intervals or refractive index is a challenge
  - Would explain complacence in optics, but Allan deviations of 10^{-15} over 10^3 s rule out drift rates down to $10^{-18}\ \mathrm{s}^{-1} \sim H_0$ *only* $\Rightarrow$ data does not rule out chirp cause of Hubble shifts!

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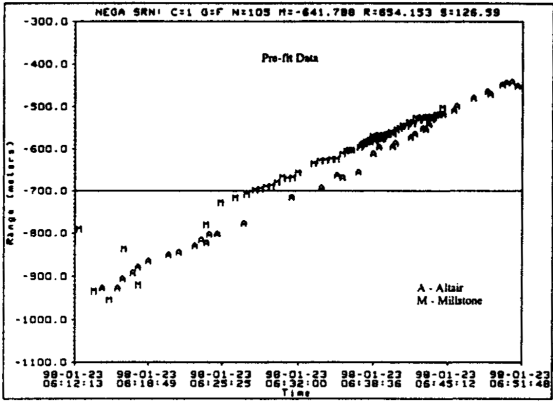
Part III

Evidence

SSN radar residuals in NEAR 1998 flyby [3]



- Negative range residuals ⇔ radar *faster* than telemetry signals

[3] Original figure from P G Antreasian and J R Guinn, AIAA, 98-4287 (1998)

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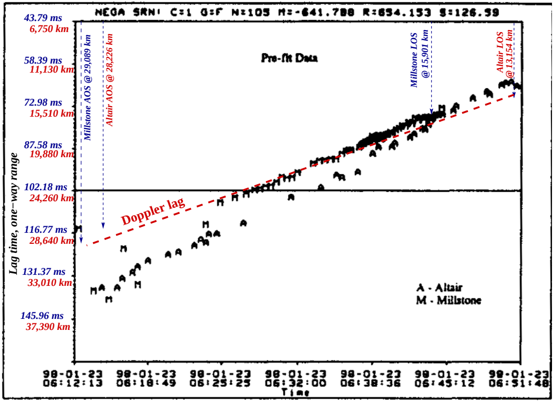
SSN radar residuals in NEAR 1998 flyby [3]



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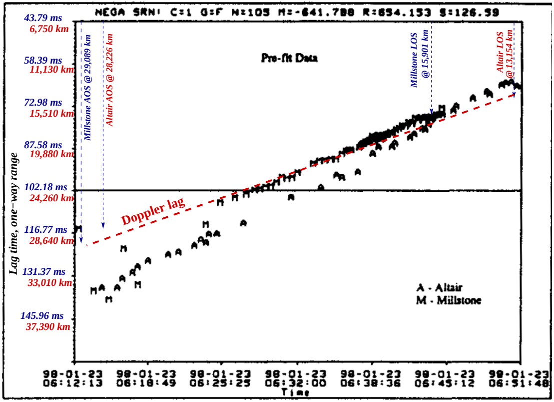
  - Residuals ∼ 1-way ranges ⇒ echoes at $2c$ OR signals at $c/2$

[3] Annotated figure from arXiv:1507.08222 EPL, 110, 5, 54001 (2015)

SSN radar residuals in NEAR 1998 flyby [3]



- Negative range residuals ⇔ radar *faster* than telemetry signals
  - Residuals $\sim$ 1-way ranges $\Rightarrow$ echoes at $2c$ OR signals at $c/2$
  - Too close for dark matter – below geostationary satellites

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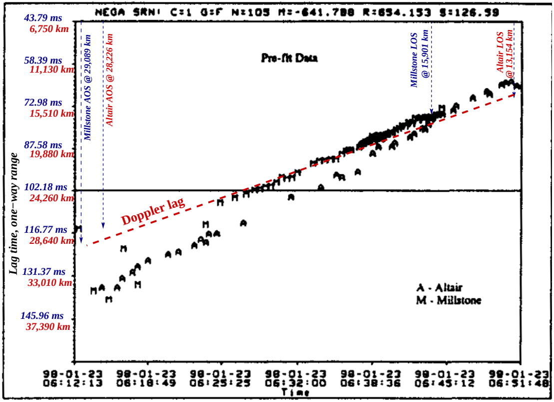

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  - Too large for relativity – over 0.01 second

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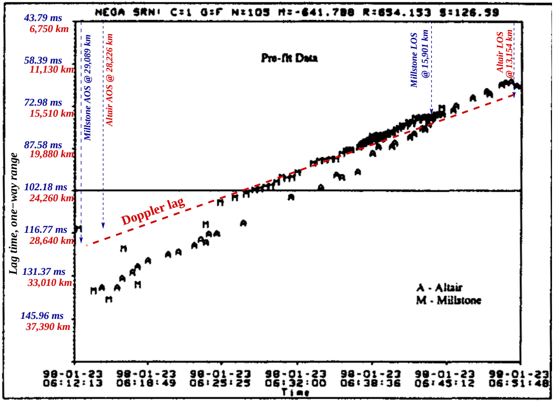

- Negative range residuals ⇔ radar *faster* than telemetry signals
  - Too large for relativity – over 0.01 second
- Occam's razor – telemetry signals were effectively delayed

[3] Annotated figure from arXiv:1507.08222 EPL, 110, 5, 54001 (2015)

The excess delays imply chirp modes

- Doppler theory for approach speed $v < c$, acceleration $a = \dot{v}$:
  - Doppler shift $\nu \to \nu(1 + v/c) \Rightarrow z \equiv \delta\nu/\nu = v/c$
  - Doppler rate $\dot{z} = \dot{v}/c = a/c$ – no delay
  - Component phases vary at rates ω
    - Phases vary with travel delay $d\phi/dt = \omega(t - r/c)$
    - Hence lags $\Delta\phi = -\omega r/c$ multi-path fading, holography

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- With chirp modes
  - Component frequencies vary at rate $\beta = \nu^{-1} d\nu/dt$
    - Frequencies vary with travel delay $\nu(t, r) = \nu e^{\beta(t - r/c)}$
    - Hence lags $\Delta\nu \approx -\nu\beta r/c$ – bats, CW-FM radar
  - Approach means r decreases at rate $v = \dot{r}$, decreasing lags
    - Explains slope of the SSN residuals
  - Lags $\Delta\nu$ means Doppler *rate* delayed by the Doppler shift!
    - Chirp carrier, information delayed – price for parallax

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Chirp mode delay explains flyby anomaly

- Pre-/post-encounter mismatch (NEAR, Galileo I, Rosetta I)
  - Theory: $\Delta\nu = 2\nu v/c - 2\nu(ar/c)/c \Rightarrow$ error $\Delta v = -ar/c$
  - Measured: $730, 760\ \mathrm{mHz} \sim 12.9, 13.5\ \mathrm{mm\ s}^{-1}$ [4]
  - Explained as of June 2016: $11.7\ \mathrm{mm\ s}^{-1}$ [5]

- Decaying diurnal Doppler tail oscillations (NEAR,Galileo)
  - Decay explained: $\Delta v = -ar/c = -GMm/rc \Rightarrow \Delta v \propto 1/r$
  - Magnitude reported (NEAR): $50\ \mathrm{mHz}$
    - Asymptote angle (-71.96°) too large, yields only $15.6\ \mathrm{mHz}$
    - The smaller pre-encounter angle -20.76° yields $47.25\ \mathrm{mHz}$
    - Some of the reported value is direction estimation error [6]

[4] From Antreasian and Guinn (1998) and Anderson *et al.* (2008), respectively.

[5] With corrections for actual AOS ground station range, courtesy J K Campbell (JPL, retired). One possibility for the remaining 1.2-1.8 $\mathrm{mm\ s}^{-1}$ is trajectory perturbation due to earth's motion during the 207 ms delay at LOS (pre-encounter), of about 6.2 km.

[6] J D Anderson *et al.*, Anomalous Orbital-Energy Changes..., PRL, 100, 9, 091102 (2008)

Converse results also explained

- Absent/negative in close watch: Galileo II, Cassini, Rosetta II
  - Excess delay $\propto$ range, should be very small near periapse
  - Radar Doppler changes sign before DSN Doppler
    $\rightarrow$ negative at periapse
- Absent with Doppler taken from FFT: Rosetta III
  - ESA uses FFT to compute carrier, Doppler – always sinusoidal
  - Rosetta III (2009) ESA tracked: $|$anomaly$|$ $<$ FFT precision [7]
  - Rosetta I (2006) was tracked by DSN and had anomaly [8]
- Newer transponder designs in Juno and later [9]
  - Digital PLLs, discretely varied oscillators
  - No more chirps – no more anomaly – problem fixed!!

[7] T Morley, https://web.archive.org/web/20120327190538/

[8] T Morley and F Budnik, Rosetta Navigation at its First Earth-Swingby, 19th Intl Symp Space Flight Dynamics, ISTS 2006-d-52 (2006)

[9] S Ciarcia *et al.*, MORE and JUNO Ka-band transponder design, performance, qualification and in-flight validation, 6th ESA Intl Wkshop Tracking, Telemetry & Command Systems for Space App (2013) paper. NASA using SDST since 1998.

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Status and opportunity

- Science completed
  - Editor's Choice paper in European Physics Letters (2015)
  - Data from STRATCOM radars and NASA/ESA tracking
  - Explained the "flyby anomaly"
- Implementation
  - Simple change to receiver frontends
  - All competition is transmit side: more power, less flexible
  - Main cost- RF designers with current skills
- World-wide patents portfolio
  - Implementatior/usage patents issuing/issued
  - PCT application for "chirp modes" filed 2015
- Status
  - Seeking university hookups for tests, development
  - Inviting licensing proposals

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