

# Prediction of spectral shifts proportional to source distances by time-varying frequency or wavelength selection

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## ABSTRACT

Any frequency selective device with an ongoing drift will cause observed spectra to be variously and simultaneously scaled in proportion to their source distances. The reason is that detectors after the drifting selection will integrate instantaneous electric or magnetic field values from successive sinusoids, and these sinusoids would differ in both frequency and phase. Phase differences between frequencies are ordinarily irrelevant, and recalibration procedures at most correct for frequency differences. With drifting selection, however, each integrated field value comes from *the sinusoid of the instantaneously selected frequency at its instantaneous received phase*, hence the waveform constructed by the integration will follow the drifting selection with a *phase acceleration* given by the drift rate times the slope of the received phase spectrum. A phase acceleration is literally a frequency shift, and the phase spectrum slope of a received waveform is an asymptotic measure of the source distance, as the path delay presents phase offsets proportional to frequency times the distance, and eventually exceeding all initial phase differences. Tunable optics may soon be fast enough for realizing such shifts by Fourier switching, and could lead to pocket X-ray devices; sources continuously variable from RF to gamma rays; capacity multiplication with jamming and noise immunity in both fibre and radio channels, passive ranging from ground to deep space; etc.

## 1. INTRODUCTION

Hitherto, there has been no specific use or test of the continuity of the Fourier spectrum. As stationary states, even conduction band energy levels in metals and semiconductors are merely quasicontinuous. The travelling wave energies are assumed to form a continuum in scattering and photoelectricity theories, but only a quasicontinuous subset actually interacts with metallic or semiconductor states, as represented by quantum interaction matrices of mixed cardinality. The Doppler effect and the Lorentz transformation have been likewise formulated primarily in terms of their impact on individual pure tones present in the spectrum. Since all observations are necessarily of finite precision, specific use of spectral continuity requires involving a rate of continuous drift of frequency or wavelength as a factor. However, though spectrometer drift is routinely corrected by recalibration and has been especially well studied in astronomical instrumentation, the observational impact of a continuously changing frequency or wavelength selection has not been specifically examined. The principal result presented here is that any frequency selective device with an ongoing drift of selection will cause received spectra to be variously and simultaneously scaled *in proportion to their source distances* to subsequent detectors. This is remarkable as the Hubble redshifts of astronomy are the only known instances of frequency shifts indicative of source distances. As a general effect of tunable or “Fourier switching” optics, it promises to add a whole new dimension in technology.

The reason is that the subsequent detectors will integrate instantaneous electric or magnetic field values from the successive sinusoids instantaneously presented by the drifting selector, and these sinusoids would differ in both frequency and phase. Current recalibration procedures, including those used for the space telescopes, were designed to at most correct for frequency variation, as integration across frequencies was never considered. Under a changing selection, however, each successive integrated field value comes from *the sinusoid of the instantaneously selected frequency at its instantaneous received phase*. The waveform constructed by the integration will therefore follow the changing selection with a *phase acceleration* defined by the rate of change times the slope of the received phase spectrum. Phase acceleration defines frequency shifts, and the phase spectrum slope is a direct asymptotic measure of the source distance, since the total path delay presents phase offsets proportional to frequency times the distance, and the offsets would eventually exceed all initial phase differences. The shifts would be therefore proportional to individual source distances in all observations with a changing frequency or wavelength selection!

The slowness of any uncorrected residual drifts and the enormous speed of light together keep this frequency shift error undetectably small for ground sources. However, at the path delays and distances of cosmology, even

drifts on geological time scales should cause errors comparable to the Hubble redshifts. Such drifts would occur from ordinary creep in metals and glasses, which is currently uncorrected for except under enormous stresses or high temperatures. The shifts for local calibration referents, used for both laboratory spectrometers and space telescopes, would be undetectable owing to the small distance. I show below that these shifts exactly fit both the cosmological acceleration and time dilation, and even satisfy Tolman's test [1, 2] for the reality of the expansion!

The very notion of exploiting phase differences across wavelengths or frequencies is generally new. Holography, synthetic aperture imaging and interferometry all involve phase differences at individual wavelengths. Pulse radar imaging exploits phase differences across the frequency comb arising from the pulse repetition, but only indirectly via a Fourier inversion [3]. The spectral phase profile must be characterized in ultrashort laser pulse technology, but only for the subsequent pulse shaping use. Optical devices are expected to soon reach tuning speeds sufficient to realize these shifts on earth, and their validation would have immense theoretic and practical implications.

The distance proportionality makes them the general time domain analogue of spatial parallax. They would enable separation of signals from cochannel transmitters by transmitter distance and instant range determination by physical means independently of the signal content, in analogy to the known angular separation of incoming signals and the instant determination of transmitter bearing using directional antennae. They would thus enable arbitrary band-limited communication simultaneously between arbitrary pairs of locations, by physics instead of modulation and multiplexing in time or frequency domains. This would not only help realize Shannon's vision of simultaneous point-to-point communication [4], but would multiply channel capacities by their concurrent reuse by noncolocated transmitters. They could be used independently, or in combination with all current technologies like TDMA, FDMA, CDMA or WDMA in air, space or optical fibres, for multiplying bandwidths. The physical plane separation of signals by source distance would also endow receivers with inherent immunity from in-channel noise and jamming. They would additionally simplify passive radar and sonar by obviating computation intensive path time correlations with multiple sources, and enabling aperture synthesis without phase reference.

Another class of potential applications for the proportional shifts is as frequency and wavelength transformers. Transformers could be constructed to turn radio, terahertz or infrared wavelengths into visible, or visible into ultraviolet or x-ray wavelengths, or the other way around, with accuracy and continuous tunability. Wavelength transformation or scaling could be used both for imaging and diagnosis, by scaling received wavelengths to values more suitable for observation and analysis, and for synthesis or generation, by scaling the wavelengths of available sources to values desired in any particular application. For example, we might obtain coherent x-ray beams by scaling optical lasers, or high power coherent infrared or optical beams by scaling up microwaves.

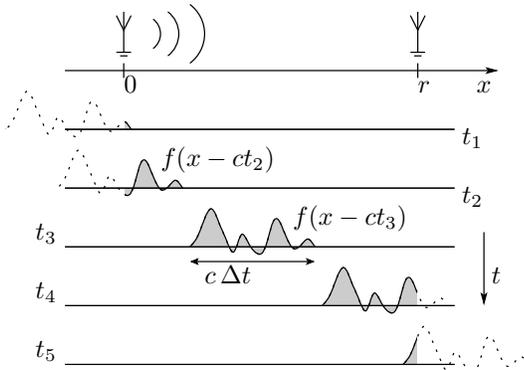


Figure 1. Travelling waveform

The rest of this paper is organized as follows. The continuity of real travelling wave spectra is established in Section 2, as we propose practical use of it for the first time. Section 3 shows that the spectral continuity implies the presence of full path length information in received spectra in the form of the slope of phase spectrum. Section 4 describes how this path length information can be transformed into innovative spectral shifts by continuous change of the instantaneous frequency or wavelength selections in a receiver or observing instrument. Sections 5-7 discuss the applications of the shifts and a theoretical verification. Broader implications including time dilation in the Doppler effect, and to cosmology are briefly discussed in Section 8.

## 2. CONTINUITY OF TRAVELLING SPECTRA

Transmission of information or energy by a travelling waveform requires that the waveform begin at a definite instant at a source location and end at a later instant at a receiver location. Fig. 1 illustrates the idea. A finite real-valued waveform  $f(x - ct)$  is shown starting at instant  $t = t_1$  at a source located at the origin  $x = 0$ , and propagating all the way to a receiver at  $x = r$ . The waveform is shown at successive instants  $t_1, t_2, \dots, t_5$  when it arrives at the receiver. The ordinary notation for a travelling waveform as  $f(x \pm ct)$  does not expressly limit its existence to a total interval of  $r/c + \Delta t$ , where  $r$  is the distance

between the source and the receiver and  $\Delta t$  denotes the entire duration of the waveform at the source or at the receiver. Without this express limitation, the waveform appears to continue to exist for  $t < t_1$  and  $t > t_5$ . The usual portrayal of signals as time-varying functions also makes it hard to notice that a travelling waveform is equivalently a spatial function  $f(x)$  with a finite compact support  $[0, c\Delta t]$ .

In physical terms, a discontinuous spectrum means that either the spectrum contains intervals of zero amplitude, or has abrupt changes in amplitude, so that either the amplitude slope or the phase slope is undefined or diverges to  $\pm\infty$  at one or more points. The continuity is needed for the existence of the phase spectrum slope, on which the innovative method of Section 4 depends, and is assured by the following basic result from Fourier theory.

**LEMMA 2.1 (SPECTRAL CONTINUITY).** *The Fourier spectrum of any waveform of finite duration or extent must be continuous and infinite.*

*Proof.* We can impose a finite compact support  $[0, T]$  upon an arbitrary waveform  $g(x)$ , by multiplying  $g(x)$  with the interval function

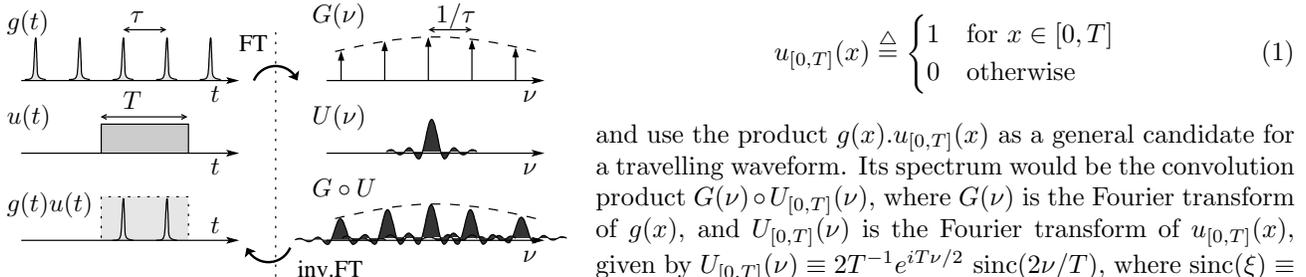


Figure 2. Spectral continuity

and use the product  $g(x).u_{[0,T]}(x)$  as a general candidate for a travelling waveform. Its spectrum would be the convolution product  $G(\nu) \circ U_{[0,T]}(\nu)$ , where  $G(\nu)$  is the Fourier transform of  $g(x)$ , and  $U_{[0,T]}(\nu)$  is the Fourier transform of  $u_{[0,T]}(x)$ , given by  $U_{[0,T]}(\nu) \equiv 2T^{-1}e^{iT\nu/2} \text{sinc}(2\nu/T)$ , where  $\text{sinc}(\xi) \equiv \sin(\xi)/\xi$  is continuous and infinite. Even if  $G(\nu)$  itself were discontinuous, say as a set of impulses,  $U(\nu)$  would make the convolution product  $G \circ U$  continuous and infinite, as shown for a pulse train over time in Fig. 2.  $\square$

The requisite continuity of the phase spectrum is thus assured for travelling waveforms on grounds that *their finite duration makes their spectra continuous and infinite*. For ranging applications, it suffices to have a finite, well-defined phase spectrum slope at one or more frequencies. For communication and wavelength transformation applications, the slope must exist over the entire spectrum, barring an at most countable subset of frequencies, whose contribution in the inverse Fourier transform would be vanishingly small.

In Fig. 2, we have also accounted for the finiteness of real pulse widths, which imposes an overall sinc profile (broken line) on the spectrum. This outer profile limits the bandwidth in digital communication, since a narrower bandwidth would widen pulses and reduce the bit rate. The inner sinc profiles from the duration of transmission have never needed consideration in the past as they overlap the thermal linespreads of natural sources and noise and clock jitter in communication. The method of Section 4 extracts coherent information from these linespreads.

### 3. SOURCE LOCALIZATION BY DIFFERENTIAL PATH PHASE

Lemma 3.1 below explains how even strictly periodic waveforms can transmit absolute source distance information. This is ordinarily quite unintuitive, and depends on being able to choose infinitesimally close pairs of wavelengths. Lemma 3.2 establishes more particularly that the slope of the phase spectrum gives a direct measure of the source distance, and is the actual basis of the method of Section 4. The spectral continuity property established above guarantees the existence of both infinitesimally close wavelength pairs and phase spectrum slopes.

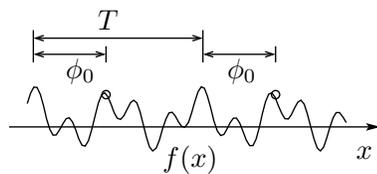


Figure 3. Periodic waveform

Both lemmas are more generally applicable to periodic waveforms, which cannot of themselves represent travelling waveforms. However, decomposition by the Fourier transform leads to sinusoidal components which are periodic, and other periodic functions are often used in signal processing. The intuition in Lemma 3.1 is that *the wavelength of the beat wave between infinitesimally close wavelength pairs is itself infinite, which makes the beat wave aperiodic*.

Fig. 3 illustrates how a known initial phase identifies the possible locations of a wave source, and thus partially determines its absolute location. Consider

a function  $f(x)$  which is periodic with the period  $\lambda$ , i.e.,  $f(x+n\lambda) = f(x)$  for integral values of  $n$ . The generalized phase  $\phi$  of  $f$  at an arbitrary point  $x$  would be simply the scaled normalized value  $\phi_f(x) = 2\pi(x \bmod \lambda)/\lambda$ , relative to the origin, which we may choose, without loss of generality, as the location of our receiver. If we determine the form of  $f$ , and its phase at the receiver location, we would be able to compute  $f$  at any offset  $x$  in the direction of propagation. If  $\phi_0$  denotes the known initial phase of the periodic component, in the overall decomposition, then the set of the possible locations of the source is  $\mathcal{L}_\lambda(\phi_0) = \{\dots, (-1 + \phi_0/2\pi)\lambda, \phi_0\lambda, (1 + \phi_0/2\pi)\lambda, (2 + \phi_0/2\pi)\lambda, \dots\} \equiv \{(n + \phi_0/2\pi)\lambda, n \in \mathbb{Z}\}$ , where  $\mathbb{Z}$  is the set of integers.

Another useful quantity is the density of  $\mathcal{L}_\lambda$ , which is simply  $\lambda^{-1}$ , because the lower the density, the less ambiguity we would have in locating the source within a given coordinate range. For example, if we knew by another means that the source might be in the coordinate interval  $[a, b)$ , the set of possible locations could be reduced, by a robust determination of the phase of  $f$ , to  $\mathcal{L}_\lambda \cap [a, b)$ . *The ambiguity in localizing the source would be  $\lceil L/\lambda \rceil - 1$ , where  $L \equiv |[a, b)|$  represents the length of the interval.* The source would be perfectly localized in the distance interval  $[a, b)$  if the ambiguity were zero. Incidentally, as accurate determination of phase is difficult, and exact analogue comparison of a real valued quantity is in any case impossible, we should more precisely represent the set of possible locations in terms of  $\phi_0 \pm \delta\phi/2$ , where  $\delta\phi$  is the uncertainty in the phase measurement. However, this uncertainty is actually inconsequential as the ambiguity comes from repetitions of the wave period. Consider that when  $f$  is a sinusoid,  $\phi_0 = 0$ , and  $\delta\phi = \pi$ , each positive (or negative) “lobe” of the sinusoid identifies an interval likely containing a point source (Fig. 4, top). If we improved the phase resolution to select a quarter cycle, as shown at the bottom of Fig. 4, we would have obtained 2 bits of information to locate the source within each cycle, but would not have identified its specific cycle. *The ambiguity in terms of localizing the source down to a specific cycle is thus independent of the precision with which phase can be determined.*

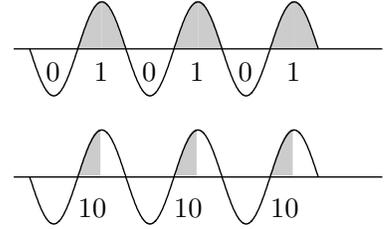


Figure 4. Localization information

Any periodic function would similarly have an infinite ambiguity in terms of localizing the source to a specific cycle. However, an arbitrary linear combination of periodic or sinusoidal functions need not be periodic. If the periods  $\lambda_1$  and  $\lambda_2$  of a pair of periodic functions are given to be rationally related, i.e., there exist integers  $a$  and  $b$  such that  $\lambda_1/\lambda_2 = a/b$ , then a combination of the two will have the period  $ab\lambda$ , where  $\lambda$  is defined by  $\lambda_1 = a\lambda$  and  $\lambda_2 = b\lambda$ . By the same reasoning, a combination of functions of *irrationally related* periods cannot be periodic. Since rationally related periods constitute only a dense subset of the 2-D real number continuum  $\mathbb{R}^2$ , the availability of absolute source distance information from a truly continuous spectral decomposition cannot be ruled out! However, if extracting information from the *nowhere dense* subset of irrational period pairs looks tricky, the following trigonometric identities

$$\begin{aligned} \sin(k_1x) + \sin(k_2x + \theta) &= 2 \sin\left(\frac{[k_1 + k_2]x + \theta}{2}\right) \cos\left(\frac{[k_1 - k_2]x - \theta}{2}\right) & [k_i \equiv 2\pi/\lambda_i, i = 1, 2] \\ \text{and } \cos(k_1x) + \cos(k_2x + \theta) &= 2 \cos\left(\frac{[k_1 + k_2]x + \theta}{2}\right) \cos\left(\frac{[k_1 - k_2]x - \theta}{2}\right) \end{aligned} \quad (2)$$

show that combinations of sinusoids (of equal amplitudes) are *always* periodic. The solution is to use beat waves.

LEMMA 3.1 (SOURCE LOCALIZATION). *Given a wavelength  $\lambda$  in the Fourier spectrum of a travelling waveform  $f(x)$ , and given a real number  $L \gg \lambda$ , a second wavelength  $\lambda'$  exists in the spectrum such that the source can be located without cyclic ambiguity in the direction of travel within any interval of length  $|L|$  containing the source, by measuring the phase of a combination of these Fourier components at an arbitrary distance from the source.*

*Proof.* From Fig. 4, the ambiguity of localizing to a cycle in the interval  $L$  with a sinusoid of period  $\lambda$  is

$$\text{ambig}(L, \lambda) = \lceil L/\lambda \rceil - 1 \quad . \quad (3)$$

By equations (2), the beat wave between sinusoids of periods  $\lambda$  and  $\lambda'$  would have the period  $\lambda'' \equiv |\lambda^{-1} - \lambda'^{-1}|^{-1}$ , with the corresponding cyclic ambiguity  $\text{ambig}(L, \lambda'')$ . We would therefore have

$$\begin{aligned} \text{ambig}(L, \lambda'') < \text{ambig}(L, \lambda) &\Leftrightarrow \lceil L/\lambda \rceil < \lceil L/\lambda'' \rceil \Leftrightarrow \lambda'' > \lambda \\ &\Leftrightarrow |\lambda^{-1} - \lambda'^{-1}|^{-1} \equiv \frac{\lambda'\lambda}{|\lambda' - \lambda|} > \lambda \Leftrightarrow \lambda' > |\lambda' - \lambda| \quad , \end{aligned} \quad (4)$$

i.e., we can reduce this ambiguity by simply choosing  $\lambda'$  close enough to  $\lambda$ , and making sure the  $\lambda'$  component has amplitude close to that of  $\lambda$  so that equations (2) can be applied. By Lemma 2.1, the spectrum of a travelling waveform must be both continuous and infinite. Hence, we can in principle choose  $\lambda'$  arbitrarily close to  $\lambda$ , and be also assured of equal amplitude. Doing so would make  $\text{ambig}(L, \lambda'')$  arbitrarily small, including zero, since

$$\text{ambig}(L, \lambda'') < 1 \quad \Leftrightarrow \quad 1 < \left\lceil \frac{L}{\lambda''} \right\rceil \quad \Rightarrow \quad \frac{\lambda' \lambda}{|\lambda' - \lambda|} \geq L \quad (5)$$

i.e., close enough to  $\lambda$ , provided  $L < \infty$ . We would also need a way to determine the initial phase value  $\phi_0$  of the beat wave. The continuity of the travelling waveform spectrum permits a reasonable assumption that the initial phases of adjacent sinusoids would be close as well. That is, we may assume the initial phase offset  $\theta \rightarrow 0$  in equations (2) as  $\lambda' \rightarrow \lambda$ . Then, regardless of what phases the sinusoids started with at the source, their initial phase difference, which gives the initial phase  $\phi_0$  of their beat wave, must be zero. A single measurement of the beat wave phase thus suffices to uniquely determine the source location within  $L$ .  $\square$

Lemma 3.1 only establishes the existence of absolute source distance information in a travelling wave spectrum. It is not suitable for practical use, however, as it calls for precise selection of a pair of wavelengths  $\lambda$  and  $\lambda'$  that must be distinct but very close to each other. Rearranging inequality (5), we get

$$\lambda < \lambda' \leq \lambda(1 - \lambda/L)^{-1} \approx \lambda(1 + \lambda/L) \quad . \quad (6)$$

The tightness of this bound is dictated by the useful choices for  $\lambda$  and expected range  $L$ . For optical wavelengths  $\lambda \sim 10^{-7}$  m, even a laboratory range of  $L \sim 1$  m imposes a bound of  $\lambda' \leq \lambda(1 + 10^{-7})$ . This is certainly useless for terrestrial or astronomical distances, as even for  $L = 10$  km, we would need a wavelength resolution of  $10^{-11}$ . This is complementary to the problem of measuring phase to some number of bits of precision in the first place, as more than one bit of precision would be needed if we chose  $\lambda$  comparable to  $L$  to simplify the pair selection.

An alternative to Lemma 3.1 comes from the theory of Green's functions, which are solutions of

$$LG(x, s) = \delta(x - s) \quad , \quad (7)$$

where  $L$  is a linear operator acting on  $x$ , and  $\delta(x - s)$  represents the Dirac delta. When this is multiplied on both sides by  $f(s)$  and integrated over  $s$ , the result is

$$\int LG(x, s)f(s) ds = \int \delta(x - s)f(s) ds = f(x) \quad .$$

As  $\int LG(x, s)f(s) ds \equiv L \int G(x, s)f(s) ds$ , this enables solution of  $Lu(x) = f(x)$  via  $u(x) = \int G(x, s)f(s) ds$ . In diffraction theory, boundary value problems are solved using  $L = \nabla^2$ , the Laplacian operator, so that  $\nabla^2 G(x, s) = \delta(x, s)$ . A general form of the boundary value problem is given by Green's second identity  $\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot d\vec{\sigma}$ , where  $\vec{\sigma}$  is everywhere normal to the boundary surface  $S$ , and is solved by setting  $\psi = G$ . Trial solutions are applied of the form  $\psi = s^{-1} e^{iks}$ , where  $s$  denotes distance from the source to the point of integration (see [5, Chapter 8.3]). These trial functions are Green's functions corresponding to a point impulse source and embody Fourier decomposition, as  $k \equiv 2\pi/\lambda$  denotes the wave number of sinusoid. The generality of this approach stems from the equivalence of an arbitrary source to a space-time distribution of point impulses, so that the electromagnetic potential distribution and wave propagation become integrals of Green's function solutions over all such source distributions.

An inseparable property of these Green's functions, which hitherto had little significance of its own, is that *all of the component sinusoids would have the same starting phase of zero*, as the phase factor  $e^{iks}$  in the Green's functions does not include a phase offset. This is more general than the zero starting phase of differential beat waves, and more particularly implies that the phase offsets of the Green's function solutions at a distance  $x$  from the point source would be equal to  $kx$ , and hence proportional to frequency, since  $\omega = kc$ . As a result, the slope of the phase spectrum would be proportional to  $x$ , as illustrated in Fig. 5, which leads to the next result.

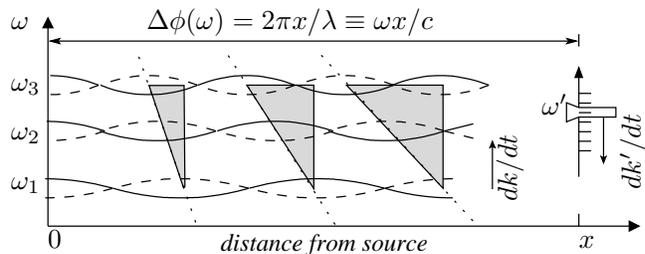


Figure 5. Spectral phase gradients

LEMMA 3.2 (PHASE SPECTRAL SLOPE). *Given a travelling waveform  $f(x)$ , the distance to its point of origin is well indicated by the slope of its phase spectrum at large distances from the origin.*

*Proof.* By definition, the Fourier decomposition of  $f$  and its inverse are

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx} dx \quad \text{and} \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k) e^{ikx} dk \quad . \quad (8)$$

The inverse transform more particularly expresses the travelling waveform as the sum of its spectral components  $F(k) e^{ikx}$ . In signal processing theory, the phase of an arbitrary complex-valued signal  $z(t) \equiv r(t)e^{i\phi(t)}$  is defined as  $\phi(t) = \arg(z(t)) \equiv -i \log(z(t))$  restricted to  $(-\pi, \pi]$ . Hence, the slope of the phase spectrum is the gradient

$$\frac{\partial \phi}{\partial k} \equiv \frac{\partial}{\partial k} [i^{-1} \log(F e^{ikx})] = i^{-1} \frac{\partial}{\partial k} [\log F + ikx] = x + i^{-1} \frac{\partial \log F}{\partial k} \quad . \quad (9)$$

The continuity of the spectrum, which is assured only for travelling waves, is necessary for defining the gradient. The derivative last term in equation (9) comprises the starting phase variations, and is therefore constant. As a result, its relative contribution would diminish with distance, i.e.,

$$\frac{\partial \phi / \partial k}{x} = 1 + \frac{\partial \log F / \partial k}{ix} \longrightarrow 1 \quad \text{as} \quad x \longrightarrow \infty \quad , \quad (10)$$

since  $F$  is independent of  $x$ .  $\square$

Lemma 3.2 is a more general result as we would not be dependent on measuring a differential beat phase, but measuring the slope still entails accurate measurement and correlation of the phase over at least two frequencies in the received spectrum. Remarkably, we can exploit the phase spectral slope without any phase measurements, so as to efficiently obtain the distances of the sources, or separate their signals, as will be explained in Section 4.

Though novel to most physicists and optical engineers and never successfully exploited before, this roll-off of phase with distance is not totally unfamiliar to signals engineers\*. The closest notion in optics is the coherence length, defined as  $L_c = \lambda^2 / \eta \Delta\lambda$ , where  $\Delta\lambda$  is the half-power (3 dB) bandwidth around the wavelength of interest  $\lambda$ , and  $\eta$  is the refractive index of the medium. Since even the most stable lasers are limited to coherence lengths of a few metres, the linespreads  $\Delta\lambda$  are known to be nonzero, and would correspond to the spectral continuity of individual wavepackets as illustrated in Fig. 2. In interferometry and holography, the term coherence refers to phase correlation over incremental distances (see [5, Chapter X]). Likewise, Doppler theory involves counting of wavefronts only of individual wavelengths, and is orthogonal to the phase variation across wavelengths considered here. This will become especially clear in the next section.

#### 4. FREQUENCY SHIFTS BY FOURIER SWITCHING

In general, a gradient  $\partial\psi/\partial y$  of a physical property  $\psi$  transforms into a rate of change  $d\psi/dt$ , if its domain is traversed at a steady speed  $dy/dt$ , since  $(\partial\psi/\partial y)(dy/dt) = d\psi/dt$ . This is especially useful in cases where  $d\psi/dt$  is easier to measure than  $\psi$  or  $\partial\psi/\partial y$ .

The phase spectrum slope  $\partial\phi/\partial k$  (equation 9) is difficult to measure, but a rate of change of phase,  $d\phi/dt$ , is generally easy, since a rate of change of phase is either a frequency or a frequency shift. The required domain traversal is a continuous variation of the receiver's frequency or wavelength scale at the rate  $dk/dt$ .

Fig. 6 depicts the various contributions to the instantaneous phase at the photodetector in an instrument equipped with a diffraction grating and a lens for discriminating frequencies or wavelengths. The total instantaneous detector phase  $\phi''$  includes temporal variation  $\omega t$  starting at the source with a static phase offset  $-\phi^s$ ; path delay from the source to the grating  $-\omega r/c$ ; an additional phase

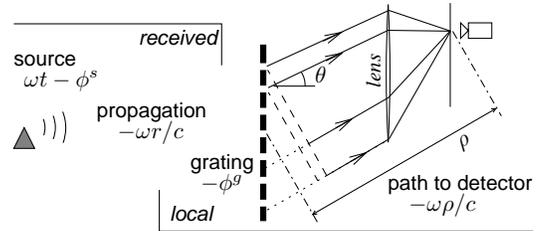


Figure 6. Components of phase

\*Dr. John A. Kosinski, US Army RDECOM CERDEC I2WD, has privately mentioned it as a long held intuition.

change  $-\phi^g$  due to transmission by the grating; and the path delay  $-\omega\rho/c$  from the grating to the detector,  $\rho$  denoting the grating-detector path length. The total instantaneous phase  $\phi''$  at the detector is thus

$$\phi'' = \omega t - \phi^s - \omega r/c - \phi^g - \omega\rho/c \quad , \quad (11)$$

so that the frequency actually detected would be

$$\frac{d\phi''}{dt} = \frac{d}{dt}(\omega t - \phi^s - \omega r/c - \phi^g - \omega\rho/c) = \omega(1 - v/c) \quad , \quad (12)$$

where  $\omega$  is the angular frequency selected by the diffraction angle  $\theta$ , and  $v \equiv dr/dt$  is the relative velocity from the source. The second term in the final result accounts for the Doppler effect. The static offset  $\phi^s$  at the source, the grating shift  $\phi^g$ , and the instrument path delay term  $\omega\rho/c$  do not have any effect on the detected frequency. The signs were chosen so that the wave phase could be written as  $\phi(r, t) = \omega(t - r/c)$ . This is the ordinary case taken for granted in current spectroscopy, and it would be straightforward to add relativistic corrections.

The frequency scale can be varied in one of two ways to exploit the phase spectrum slope  $\partial\phi/\partial k$ . We could move the detector over the focal plane of the lens, so as to select successive frequencies  $\omega'$  at a rate  $dk'/dt$ , as suggested by the detector scope and scale markings depicted at the right of Fig. 5. Or we could vary the grating intervals so as to cause the frequency  $\omega$  arriving at a fixed diffraction angle  $\theta$  to vary at the rate  $dk/dt$ . In either case, *the detector would be presented a continuous sequence of instantaneous values from a succession of Fourier (sinusoidal) components of the travelling waveform, in place of a single Fourier component*, so that the phase at the detector would be accelerated relative to the static case in equation (12), with the following result.

**THEOREM 4.1 (PHASE ACCELERATION).** *To an instrument with frequency scale changing at a normalized rate  $\beta$ , the spectra of static sources will appear scaled in proportion to the source distances  $r$  as  $\omega'' \simeq \omega(1 + \beta r/c)$  or, equivalently, at scale factors  $z(r) \equiv \delta\omega/\omega \equiv (\omega'' - \omega)/\omega \simeq +\beta r/c$  at  $r \gg \lambda$ .*

*Proof.* Assuming, as in Lemma 3.1, that the amplitude varies slowly in the arriving travelling wave spectrum, the variation of phase  $\phi''$  at the detector will be primarily due to the phases of the Fourier components and other phase contributions already noted in equation (11), and not so much from the amplitude differences between the Fourier components. Then, to a first order, the rate of change of phase at the detector would be

$$\frac{d\phi''}{dt} = \left[ \frac{d\phi}{dt} - \frac{d\phi^s}{dt} - \frac{d(\omega r/c)}{dt} \right] - \left[ \frac{d\phi^g}{dt} + \frac{d(\omega\rho/c)}{dt} \right] \quad (13)$$

where the first three terms on the right refer to the rate of change of phase entering the grating, and the remaining two, the rate of change due to the grating itself. We have rewritten  $\omega t$  as  $d\phi/dt$  to allow for the fact that  $\omega$ , representing the instantaneously selected Fourier component, would be itself changing.

We shall represent the instantaneous selection by a single prime, as  $k'$ . The first term on the right in equation (13) then leads to two parts, the intrinsic rate of change due to the original angular frequency  $\omega$ , and a part reflecting the changing wavelengths at the same diffraction angle  $\theta$ , as

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial k'} \frac{dk'}{dt} \equiv \omega + \dot{k}' \frac{\partial\phi}{\partial k'} = \omega \quad , \quad (14)$$

since diffraction *per se* does not contribute to phase nor, hence, to its rate of change; dispersion by the grating will be accounted for separately below by terms in  $\phi^g$ . The second term on the right in equation (13) expands similarly, except  $\partial\phi^s/\partial t \equiv 0$  as  $\phi^s$  are constants with respect to time. However,  $\dot{k}' \partial\phi^s/\partial k'$  survives because  $\phi^s$  would likely vary across wavelengths, and hence across successive values of  $k'$ . Modulation at the source would contribute to variation of  $\phi^s$  across wavelengths, and would be hence contained in this term. For constant wave speed  $c$ , the remaining terms similarly expand to

$$\begin{aligned} \frac{d(\omega r/c)}{dt} &= \dot{r} \frac{\partial(kr)}{\partial r} + \dot{k}' \frac{\partial(kr)}{\partial k'} = k\dot{r} + \dot{k}'r \quad , \\ \frac{d\phi^g}{dt} &= \frac{\partial\phi^g}{\partial t} + \dot{k}' \frac{\partial\phi^g}{\partial k'} \quad \text{respectively, and} \\ \frac{d(\omega\rho/c)}{dt} &= \dot{k}' \frac{\partial(k\rho)}{\partial k'} = \dot{k}'\rho \quad , \end{aligned} \quad (15)$$

noting that  $k \equiv \omega/c$  generally, and  $\dot{\rho} = 0$  for a static lens assembly. Also,  $\partial\phi^g/\partial t$  cannot vanish while the grating is being varied, but as remarked above, we could move the photodetector instead and keep the grating fixed, in which case, we would indeed have  $\partial\phi^g/\partial t = 0$ . In either case, equation (13) leads to

$$\frac{d\phi''}{dt} \equiv \omega'' = \omega - k\dot{r} - \dot{k}'(r + \rho + \phi_{,k'}^s + \phi_{,k'}^g) \quad (,k' \equiv \partial/\partial k') \quad (16)$$

Absorbing  $\rho$  into  $r$ , replacing  $\dot{r}$  with  $v$ , and noting that  $k' \equiv k$  because the output and input wave vectors at the grating would be identical at each instant, we get

$$\begin{aligned} \omega'' \equiv \dot{\phi}'' &\equiv \omega(1 - v/c + [r + (\phi_{,k}^s + \phi_{,k}^g)]\beta/c) \simeq \omega(1 - v/c + \beta r/c) \quad , \\ \text{and } z(r) \equiv \delta\omega/\omega &\equiv (\omega'' - \omega)/\omega = -v/c + [r + (\phi_{,k}^s + \phi_{,k}^g)]\beta/c \simeq -v/c + \beta r/c \quad . \end{aligned} \quad (17)$$

The asymptotic form generally holds for  $r \gg \lambda$ , since the source phase modulation and grating delay variation would be unlikely to exceed a few wavelengths. To prove that the spectrum would be indeed scaled by the factor  $(1 - v/c + \beta r/c) \equiv \Delta$ , it is convenient to use the bra-ket notation of quantum mechanics. Fourier decomposition in general is governed by the orthogonality condition,  $\langle \omega'' | \omega, r \rangle \equiv \int e^{-i\omega'' t} e^{-i(kr - \omega t)} dt = e^{-ikr} \delta(\omega'' - \omega)$ . As  $t$  literally denotes time *measured by the Fourier analyzer clock*, the orthogonality should allow for a variability in the ‘‘ticking’’ of  $t$ , and thus more generally written as  $\langle \omega'' | \omega, r \rangle \equiv \int e^{-i\omega'' t} e^{i(d\phi''/dt)(t-r/c)} dt$ . Identifying  $d\phi''/dt$  as the phase at the detector, and applying the asymptotic forms in equations (17), we get

$$\begin{aligned} \langle \omega'', \beta | \omega, r \rangle &\equiv \int e^{-i\omega'' t} e^{i(d\phi''/dt)(t-r/c)} dt = \int e^{-i\omega'' t} e^{i\omega(1-v/c+\beta r/c)(t-r/c)} dt \\ &= e^{-i(k\Delta)r} \delta(\omega'' - \omega\Delta), \quad \Delta \equiv (1 - v/c + \beta r/c) \quad . \end{aligned} \quad (18)$$

For an arriving waveform  $|f, r\rangle \equiv f(t-r/c)$ , the notation  $\langle \omega | f \rangle \equiv F(\omega)$  yields  $\langle \omega'' | f, r \rangle \equiv F(\omega'', r) = e^{-ikr} F(\omega'')$ .

In the presence of phase acceleration,  $\langle \omega'' | f, r \rangle$  changes to

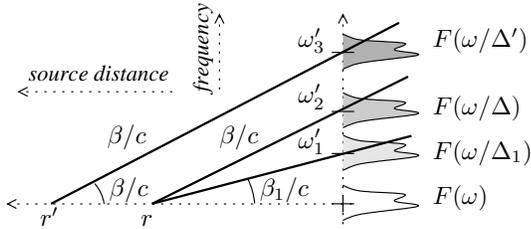


Figure 7. Time domain parallax

$$\begin{aligned} \langle \omega'', \beta | f, r \rangle &\equiv \int_t \int_\omega \langle \omega'', \beta | t \rangle dt \langle t | \omega \rangle d\omega \langle \omega | f, r \rangle \\ &= \int_\omega \int_t e^{-i\omega'' t} dt e^{i(d\phi''/dt)(t-r/c)} F(\omega) d\omega \\ &= \int_\omega e^{-ik''r} \delta(\omega'' - \omega\Delta) F(\omega) d\omega = e^{-ik''r} F(\omega''/\Delta) \quad , \end{aligned} \quad (19)$$

using equation (18) to reduce the time integral. Equation (19) proves that the scaling would be real, uniform and proportional to the source distance  $r$ , and therefore also distinct from the changing scale of the instrument. It is straightforward to prove, by applying the superposition principle to the travelling waveforms, that equations (11)-(17) would also hold for a multitude of travelling waveforms  $f_j(x)$  arriving simultaneously from sources at different distances  $r_j$ , and would yield the scale factors  $\Delta_j \simeq (1 - v_j/c + \beta r_j/c)$ , for the respective waveforms, for the same common  $\beta$ , so that the spectra would be separated in frequency according to the source distances, as illustrated in Fig. 7.

For instance, for an astronomical source at  $r/c \sim 10 \text{ Gy}^\dagger \equiv 3.156 \times 10^{16} \text{ s}$ ,  $\beta \approx 3.17 \times 10^{-17} \text{ s}^{-1}$  would suffice for  $z = 1$ , i.e., doubling in frequency. This is so small that the total change of the grating intervals  $\zeta(\Delta t) = \int_{\Delta t} \beta d\tau$  would amount to just  $1.13 \times 10^{-14}$ , or a little over a cycle at visible wavelengths, for observation times of  $\Delta t = 1 \text{ h}$ , and only  $10^{-10}$  hypothetically for a full year. Larger  $\beta$  would be needed in most practical applications, but the shifts  $\beta r/c$  would remain larger than, and distinct from,  $\zeta$  over any individual Fourier integration window.  $\square$

## 5. DISTANCE MULTIPLEXING, JAMMING IMMUNITY AND PASSIVE RANGING

The principal problem of communication engineering, as remarked in the Introduction, is facilitating independent communication between multiple pairs of locations. In terrestrial communication, these end points generally lie

<sup>†</sup>Giga year  $\equiv 10^9$  years. Geological literature also uses Ma and Ga to denote mega- and giga-annum, respectively.

in the two-dimensional space of the earth's surface, as illustrated in Fig. 8. The separation in frequency by source distance allows separation of the respective signals using the physics of space instead of modulation or content. For example, in Fig. 7, the same spectrum  $F(\omega)$  could be scaled to  $F(\omega/\Delta_1)$ , where  $\Delta \equiv 1 + \beta_1 r/c$ , or to  $F(\omega/\Delta)$ ,  $\Delta \equiv 1 + \beta r/c$  if it came from a source at distance  $r$ , depending on whether the rate of change was  $\beta$  or  $\beta_1 < \beta$ , assuming the source and the receiver are relatively static. For the same  $\beta$ , the same spectrum would get scaled to  $F(\omega/\Delta')$ , where  $\Delta' \equiv 1 + \beta r'/c$  if it came from another (static) source at distance  $r'$ . Then, with a band-pass filter, we may isolate either  $F(\omega/\Delta)$  or  $F(\omega/\Delta')$ , and recover the original signal from the corresponding source by scaling the filtered spectrum back to the original frequency band. A formalism for treating this is given next.

**COROLLARY 5.0.1 (CASCADING PHASE ACCELERATION).** *Phase acceleration can be cascaded, and the product operator representing cascading of phase accelerations forms a commutative group for small  $\beta$ .*

*Proof.* We may define a *phase acceleration operator*  $H(\beta)$  by setting  $\langle \omega'', \beta | = \langle \omega'' | H(\beta)$ , so that  $\langle \omega'', \beta | f, r \rangle \equiv \langle \omega'' | H(\beta) | f, r \rangle$ . We would then have  $H(0) = 1$ , defining the identity element, since  $\langle \omega'' | H(0) | f, r \rangle \equiv \langle \omega'' | f, r \rangle \equiv e^{-ikr} F(\omega'')$ , which preserves the received spectrum. The inverse is similarly given by  $H^{-1}(\beta) = H(-\beta)$ . From the definition  $k' \equiv 2\pi/\lambda'$  which relates  $k'$  to the instantaneously selected wavelength  $\lambda'$ , we have  $k'^{-1} dk'/dt \equiv (2\pi/\lambda')^{-1} d(2\pi/\lambda')/dt = \lambda' \cdot (-\lambda'^{-2} d\lambda'/dt) = -\lambda'^{-1} d\lambda'/dt$ , i.e., *a negative  $\beta$  implies upward scaling of wavelengths instead of frequencies.* In an all-optical system using diffractive elements for both phase acceleration and filtering, we would thus need to use negative  $\beta$  for the time domain parallax and signal separation.

Since the waveform arriving at the detector in Figs. 5 and 6 already presents a scaled wavelength or frequency, it can be subjected to a further phase acceleration using a second grating with continuously changing intervals, to which equations (16) would be applicable once again. This provides a product operation for the  $H$  operators, and the product scale factor corresponding to  $H(\beta_1)H(\beta_2)$  would be  $(1 + \beta_1 r/c)(1 + \beta_2 r/c) = (1 + [\beta_1 + \beta_2]r/c + \beta_1 \beta_2 r^2/c^2) \approx 1 + (\beta_1 + \beta_2) r/c$  for  $\beta_1, \beta_2 \ll c/r$ . This yields  $H(a\beta_1)H(b\beta_2) = H(b\beta_2)H(a\beta_1) \approx H(a\beta_1 + b\beta_2)$ , so that the product is commutative and linear for small  $\beta$ , and makes the set of  $H(\beta)$  operators a group.  $\square$

The signal selection and separation process would be described by the product operator  $H^{-1}(\beta) \tilde{G}_m H(\beta) G_b$ , where  $G_b$  is the band-pass prefilter admitting the unscaled (unaccelerated) received spectrum  $F(\omega)$  as in Fig. 7;  $H(\beta)$  is the first phase acceleration employed to spread the received prefiltered signal to  $F(\omega/\Delta)$  and  $F(\omega/\Delta')$ ;  $\tilde{G}_m$  is a band-pass “source selection” filter for selecting either  $F(\omega/\Delta)$  or  $F(\omega/\Delta')$ ; and  $H^{-1}(\beta) \equiv H(-\beta)$  is the “phase deceleration” for returning the selected scaled spectrum back to its original frequency band.

Similar notions of *space division multiple access* (SDMA) and of *space division multiplexing* (SDM) characterize multibeam antennas on satellites and in orthogonal frequency division multiplexing (OFDM). These existing notions do not include multiplexing by distance, however. The connotation of “division” in these terms is also at slight variance from the traditional notions of time, frequency or code division multiplexing or multiple access, in which the channel capacity itself becomes divided, whereas the distance division capability, like the division of the solid angle in satellite SDMA, *multiplies* the overall channel capacity, as the full bandwidth can be reused by each source. The multiplication factor is quantified as follows.

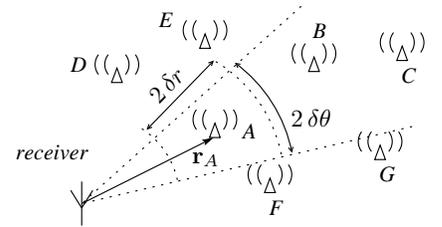


Figure 8. Multiplexing by distance

**COROLLARY 5.0.2 (DISTANCE MULTIPLEXING).** *Any receiver using phase acceleration can resolve indefinitely many band-limited transmissions at increments asymptotically proportional to the distance.*

*Proof.* Assuming a centre frequency  $f_c$  and nominal signal bandwidth  $2W$  including guard bands, a pair of sources at distances  $r$  and  $r' > r$  (Fig. 7) would be just resolved when the higher end frequency  $f_c + W$  from the source at  $r$  scales to just below the lower end frequency  $f_c - W$  from the source at  $r'$ , i.e.,  $(f_c + W)(1 + \beta r/c) = (f_c - W)(1 + \beta r'/c)$ . Rearranging and substituting  $r_n$  for  $r$  and  $r_{n+1}$  for  $r'$ , yields the infinite recursion

$$\beta r_{n+1}/c = \gamma + (1 + \gamma) \beta r_n/c \quad , \text{ where } \gamma \equiv 2W/(f_c - W) \text{ and } n = 1, 2, \dots ;$$

$$\text{whence } \delta r_n \equiv r_{n+1} - r_n = \gamma (r_n + c/\beta) \quad , \text{ so that } \frac{\delta r}{r} \simeq \gamma \simeq \frac{2W}{f_c} \quad \text{for } f_c \gg W \quad . \quad (20)$$

The result is a distance resolution of  $\pm \delta r$  proportional to  $r$ , similar to parallax.  $\square$

If combined with a directional antenna for azimuth and elevation resolutions of  $\pm\delta\theta$  and  $\pm\delta\psi$ , respectively, this would enable the receiver to selectively listen to a transmitter at any desired coordinates  $(r, \theta, \psi)$  relative to the receiver, regardless of all other transmissions on the same frequencies, friendly or otherwise, as follows.

**COROLLARY 5.0.3 (NOISE AND JAMMING IMMUNITY).** *Interference or noise originating at sufficient distance from a source of a band-limited signal can be suppressed independently of signal content, using phase acceleration.*

*Proof.* Rejection of out-of-band received interference and noise depends on the performance of the prefilter  $G_b$ . Rejection of in-band interference and noise from noncolocated sources depends mainly on the selection filter  $G_m$ . We would also expect noise and distortion due to nonuniformity and thermal or mechanical fluctuations in the diffraction grating or other means used for the phase acceleration operations  $H$  and  $H^{-1}$ . Fluctuations in  $\beta$  would introduce interference as they would cause the signals from neighbouring sources to overlap. The broad result holds for interference and noise sources outside the selected distance interval  $r \pm \delta r$ .  $\square$

Lastly, the shift  $\beta r/c$  itself is a linear instant measure of source distance, available without round-trip timing or phase correlation with another illuminating source, as required in both active and passive radars today. For visual indication, it would suffice to apply a notch prefilter to the received spectrum, and to display a precomputed distance scale alongside the frequency (or wavelength) axis with no signal correlation at all.

## 6. WAVELENGTH TRANSFORMATION AND SYNTHESIS

As  $\beta$  is a rate of change of receiver frequency scale and both  $\beta$  and the range of variation are limited only by the state of technology, phase acceleration enables continuous wavelength scaling over any range by any factor, well beyond the capability of bandgap and nonlinear materials. The scaling could be additionally used for accurately synthesizing radiation of any wavelength with desired polarization, coherence or modulation, by applying it to an available source already capable of providing the latter properties. As both  $\beta$  and the source distance  $r$  would be usable for fine control, accuracy and continuous tunability of the output wavelength are assured.

An inherent limitation lies in the fact that phase acceleration expands or contracts successive segments of the input waveform. If the wavelength is contracted, the output would contain gaps between sweeps of the frequency scale variation; if expanded, portions of the source waveform would be correspondingly skipped.

## 7. REALIZABILITY AND PARTIAL VERIFICATION

For device and terrestrial communication applications, we require  $z = O(1)$  at distance scales of under 1 m to about  $10^4$  m, which calls for normalized rates of change  $\beta \equiv zc/r \sim 10^8$  to  $10^4 \text{ s}^{-1}$ . Even for  $z \sim 10^{-6}$  at  $r \approx 1$  m, we would need  $\beta \sim 300 \text{ s}^{-1}$ . This may still seem large, but the continuous variation is only needed in repetitive sweeps that can be as short as  $1 \mu\text{s}$ . The total variation would be  $\zeta(1 \mu\text{s}) \equiv \exp(300 \text{ s}^{-1} \times 1 \mu\text{s}) - 1 \approx 0.0003$ . This is in the range of piezoelectric or magnetostrictive elements, on which a reflective plastic grating could perhaps be bonded, but the speed would be in milliseconds.

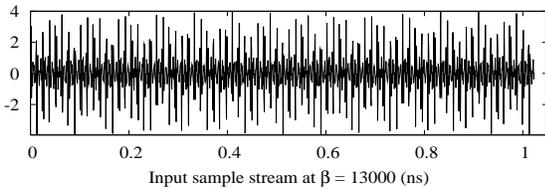


Figure 10. Distortion from variable sampling

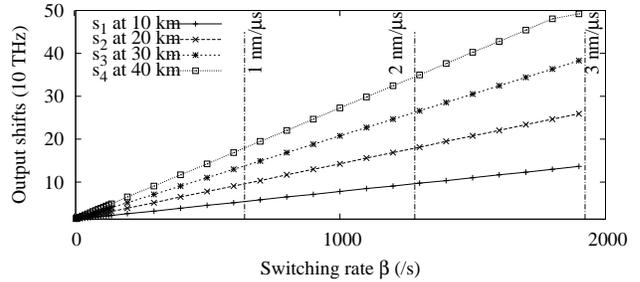


Figure 9. Computed shifts for emerging fast FBGs

Current fast-tunable fibre Bragg gratings (FBGs) that use mechanical stress are rated at switching speeds of only a few nm in wavelength per ms at around  $1.5 \mu\text{m}$ , for which  $|\beta| \sim (1.5 \mu\text{m})^{-1} \times 1 \text{ nm ms}^{-1} \approx 0.7 \text{ s}^{-1}$ . At these rates, a barely observable shift of  $z = 10^{-6}$  would need  $r \equiv cz/\beta \approx 500$  km, which makes the sun the nearest available test source! This would also explain why phase acceleration and the associated shifts remain unnoticed.

Only the next generation devices, expected to attain Fourier switching speeds in the order of nm wavelength in  $\mu\text{s}$  or  $\text{ns}^\ddagger$ , would be able to produce measurable  $z$  within a laboratory.

Fig. 9 illustrates spectral scaling and signal separation over 10 km fibre spools between four  $1.54 \mu\text{m}$  ( $\equiv 130 \text{ THz}$ ) sources, assuming a core index of  $\eta \approx 1.47$ , simulated in an equivalent digital signal processing (DSP) realization of phase acceleration using variable sampling [6, 7]. A Java applet developed for this test incorporates both deterministic and statistical simulations of linespreads in accordance with Lemma 2.1. The sampling rate variation  $\beta$  is independent of the source distances, which are input only to the path phase computation, yet the unmodified open source Java FFT applied to the sample stream reveals the shifts, as a partial test of the theory. The time domain signal is distorted by the sampling rate variation, but is still periodic, as shown in Fig. 10.

## 8. BROADER IMPLICATIONS

The principle of radiation quantization and, more particularly, Planck's quantization rule  $E = \hbar\omega$  have made it too easy to view electromagnetic radiation as comprising monochromatic quanta. This corpuscular perception should be probably blamed for the inadequate treatment of diffraction in both particle physics and astrophysics, to be pointed out below. Although the second quantization formalism is in fact based on standing wave modes, it is commonly overlooked that travelling quanta cannot be monochromatic, and therefore cannot be Planckian! Conversely, Planck quanta are necessarily merely detector state transitions, under no fundamental obligation to exactly correspond to source state transitions responsible for the radiation. The Fourier integration involved in any form of spectroscopy or frequency or wavelength selection is an inherently classical *macroscopic* process, and also under no obligation to preserve the original quanta. Although multi-quanta interactions are well known, the possibility of *reconstituting* photons by recombining multiple original Fourier components is new, and introduces a more general notion of coherent integral transformations of light. More significant implications of the present theoretic result concern astrophysics, as follows.

**PRINCIPLE 1 (HUBBLE UNCERTAINTY).** *Real spectrometers are subject to a fundamental uncertainty affecting distant spectra, given by  $\Delta\beta \Delta T_E \approx 1$ , where  $\Delta T_E$  is the time constant of component variations and  $\Delta\beta \equiv \beta$  is the resulting scale uncertainty affecting remote spectra according to equations (17). Since creep cannot be totally eliminated, taking  $t_\odot \equiv 4.9 \text{ Gy} = 1.55 \times 10^{17} \text{ s}$ , the age of the sun, as a conservative upper bound on the rigidity of our instruments, i.e.,  $\Delta T_E \leq t_\odot$ , yields a scale uncertainty of  $\Delta\beta \geq t_\odot^{-1} = 200 \text{ km s}^{-1} \text{ Mpc}^{-1} \approx 2.7 H_0$ , the Hubble constant.*

*Rationale.* Creep is usually associated with high temperatures and stresses, but solid state physics provides no fundamental constraint prohibiting dislocations even at arbitrary low stresses. The residual creep rate would be governed by the dislocation probability  $p_d \equiv e^{-W_d/k_B T}$ , where  $k_B$  is the Boltzmann constant,  $T$  is the operating temperature and  $W_d$  is the dislocation energy barrier typically about 1-2 eV, hence  $p_d \approx 10^{-11} \dots 10^{-21} \text{ s}^{-1}$ . We should expect residual creep, under the compression of earth's own gravity (due to the gradient  $\nabla\mathbf{g}$ ) and aided by the kneading effect of tides, to cause extremely slow continual shrinkage in instrument dimensions, hence a negative  $\beta$ , consistent with redshifts, of within a few orders of  $p_d$ . The Hubble redshifts are in fact of this order, since  $H_0 \approx 73 \text{ km s}^{-1} \text{ Mpc}^{-1} = 2.4 \times 10^{-18} \text{ s}^{-1}$ .  $\diamond$

Two further results reinforce this implication. First, again from equations (17), we may equate  $v = \beta r/c$  to obtain the equivalent apparent velocity of remote sources. With  $\beta \sim H_0$ , this would have the same form as the cosmological expansion. Since the motion would be apparent and not real, however, it would be perfectly linear, so that in the associated acceleration  $\dot{v} \equiv \beta\dot{r}/c + r\dot{\beta}/c = \beta^2 r/c + r\dot{\beta}/c$ , we should set  $\dot{\beta} = 0$  identically, since the acceleration refers to the optical path time (history) of the received light, and not the instantaneous variation at the instrument. In terms of relativistic cosmology, this acceleration component of the uncorrected shifts would be characterized by the deceleration coefficient  $q \equiv (-1 + \dot{\beta}/\beta^2) = -1$ . The observed cosmological acceleration in fact corresponds to  $-1 \pm 0.4$  [8]. The next result is also unmatched by alternative cosmology theories.

**THEOREM 8.1 (SPECTROMETER AND DOPPLER TIME DILATIONS).** *Both spectrometric scale drift and Doppler shifts imply a commensurate, apparent time dilation in the received light.*

<sup>‡</sup>See, for example, the Sabeus and Optonet abstracts at <http://www.dodsbir.net/selections/sttr1.05.htm>.

*Proof.* From equation (19), writing  $\tau$  for receiver time and requiring  $\langle \tau, -\beta | \omega'', \beta \rangle \equiv e^{i\omega''\tau}$  leads to

$$\begin{aligned} \langle \tau, -\beta | f, r \rangle &\equiv \int_{\omega''} \langle \tau, -\beta | \omega'', \beta \rangle d\omega'' \cdot e^{-ik''r} F(\omega''/\Delta) = \int_{\omega''} e^{i\omega''\tau} d\omega'' \cdot e^{-ik''r} F(\omega''/\Delta) \\ \text{[substituting } \omega' = \omega''/\Delta] &= \int_{\omega'} e^{i\omega'\tau\Delta} e^{-i\omega'r\Delta/c} F(\omega') d\omega' \Delta \equiv \Delta \cdot f([\tau - r/c]\Delta) \quad . \end{aligned} \quad (21)$$

This matches the cosmological time dilation, which is remarkable because time dilation has been hitherto regarded as an exclusive consequence of relativity, so much so as to attribute the Doppler time dilation also to gravity [9].

All that equation (21) represents is that a uniform scaling of frequencies means a reciprocal scaling of time. As the Doppler effect uniform scales the whole spectrum, repetition frequencies of pulses from a moving source would scale by the same amount. Time dilation looks exotic only because scattering and other matter interactions like the Wolf effect [10], do not uniformly act on the whole frequency axis, and in part because it is not significant in terrestrial applications. The next result concerns the related effect on luminosity.  $\square$

**PRINCIPLE 2 (TOLMAN'S BRIGHTNESS LAW).** *The apparent brightness of distant sources should diminish as  $\Delta^4 \equiv (1 + \beta r/c)^4 \equiv (1 + z)^4$  with distance  $r$  due to  $\beta < 0$  from any systematic scale drift in our instruments.*

*Rationale.* Tired light theory was proposed by Zwicky [11] as a possible alternative to actual expansion almost immediately following Hubble's law [12]. Tolman proposed testing the reality of expansion by verifying whether the brightness of distant sources decreased, after removing the effects of dust extinction and peculiar velocities, with the redshifts as  $(1 + z)^4$  where one factor of  $(1 + z)$  would be due to the photon energy reduction due to the redshift, a second  $(1 + z)$  factor accounts for the decrease in the photon flux rate due to the expanding distance, and the remaining  $(1 + z)^2$  accounts for larger apparent area at the time of emission (aberration), since in tired light theory, the brightness should decrease only due to redshift, as  $(1 + z)$ , as the distance expansion and area aberration factors would be absent [1, 13]. Actual tests yield exponents of 2.6 or 3.4 depending on the frequency band [14, 15, 16, 17], and the difference from the expected exponent 4 matches the brightness variation predicted in stellar evolution (cf. [18]).

The shifts due to phase acceleration would be also equivalent to the Doppler effect of apparent radial motion, corresponding to  $v/c \equiv -\beta r/c$  or  $v = -\beta r$  in equations (17), after eliminating peculiar motions. The aberration factor of  $\Delta^2 \equiv (1 + z)^2$  holds identically because when we interpret the shifts as Doppler, our resulting source models would be based on the Doppler-corrected past distances, just as assumed in Tolman's proposal, so that the observed brightnesses would have to be similarly corrected for smaller starting areas for consistent physics. Time dilation between photons would seem to be given by the second  $\Delta$  in equation (21), and it could be argued that the detector state transitions representing the detected photons would be of lower energies as well. However, we can no longer assume identity or even a 1:1 correspondence between the source emitted wavepackets and the detected photons since equation (19) signifies a *reconstitution* of photon energies. All of the preceding theory is inherently *classical*. The remaining  $\Delta^2$  factor emerges *classically* from equation (21), as the power flux is given by  $\int |\langle \tau, -\beta | f, r \rangle|^2 d\tau \equiv \Delta^2 \int |f([\tau - r/c]\Delta)|^2 d\tau$ .  $\diamond$

Though Principles 1 and 2 primarily illustrate the physics of phase acceleration and the associated frequency shifts, specifically, the amplification of creep and the redistribution of power flow, the implied challenge to current cosmology theory is unfortunately real. The elementary conjugacy of time dilation to change of frequency scale, exposed in equation (21), is also not the only omission in current mainstream cosmology.

There has been no path-integral treatment of diffraction for intra-galactic and cosmological space, as required for a volumetric distribution of absorbing bodies, which yields a propagation law of the form  $I(r) \propto r^{-2} e^{-\sigma r}$ , similar to that for the ordinary attenuation of sound. All of the diffractive considerations in both astronomy and particle physics have been limited to Fresnel-Fraunhofer approximations (cf. [19, pp.149-153], [20, pp.148-149]), which are inadequate for describing successive diffractions of wavefronts that would lead to inaccessible trapped states of energy, resembling neutrinos and contributing to both the radiation background and dark matter, as the  $r^{-1} e^{-\sigma r}$  amplitude profile is a Klein-Gordon eigenfunction for particles of effective mass  $\hbar\sigma/c$ . Diffractive (and gravitational)  $\sigma$  of just 0.05 dB Mpc $^{-1} \equiv 10^{-24}$  dB m $^{-1}$  is all it takes to yield the same cutoff of the visible universe as the redshifts (cf. [21, 22]) and diffractive loss of metallic lines due to *off-ray* proximal matter is also

unaccounted for in the treatment of primitive galaxies (cf. [23, 24]). The third key piece of the standard model, primordial nucleosynthesis, again represents a predisposed model view, since the observed nuclear abundances, such as of helium [25] or lithium [26], which are inconsistent with galactic production in the finitely old universe of current cosmology and call for a homogeneous large scale cooling, would be equally and more simply explained by accumulation in an infinitely old universe *without* requiring large scale cooling or inflation theory.

Closer to home, the measured lunar recession of  $3.82 \pm 0.07 \text{ cm y}^{-1}$  [27] exceeds a naïve linear Hubble’s law expansion on short scale<sup>§</sup>, again consistent the uniform apparent expansion of all measured distances that would be expected for a systematic instrument cause like creep. More importantly, a correction for the dilation of past time would be required in *all* measurements. Even NASA’s consideration of atomic clock drifts in the context of the Pioneer anomaly has been limited to verification of the Allan variances [35 §V-C], which are essentially autocorrelations computed by local circuits that would be similarly subject to creep. The time dilation correction would increase the sun’s age to over 7 Gy and that of the universe, to *infinity*, consistent with the above inference for nuclear abundances, and more conservative than the standard model.

## Conclusion

I have shown that a drift of scale in frequency or wavelength selection will cause phase acceleration in proportion to the slope of the phase spectrum of the received radiation, and result in scaling of subsequently detected spectra in proportion to the source distance of the radiation. The acceleration must occur because the waveform arriving at the subsequent detector comprises instantaneous values of the radiation from different successive sinusoids present in the Fourier decomposition of the original waveform. The spectral scaling must occur since the detector can only integrate the waveform actually presented to it by the preceding selection. The phase spectral slope to support the phase acceleration requires continuity of the received spectrum, which is in turn guaranteed by the finite beginning and end implicit for any wavepacket transporting information or energy. Proportionality to the source distances is assured by the total path delay from the source, proportionality of the phase represented by the fixed path delay in an individual sinusoid to its frequency, and the dominance of this path phase contribution at large distances over any initial phase differences between the sinusoids. All of the mathematical premises are basic and independent of relativity or quantum theory, and the mathematical consistency of the prediction has been verified by digital simulation of linespreads with the path delay using variable sampling to simulate receiver clock drift equivalent to a drift of its frequency or wavelength selection.

This prediction is as yet theoretic, like Doppler’s in 1842 [36]. The adequacy of real linespreads, for example of continuous wave lasers and of radio waves, for the envisaged terrestrial and noncosmological applications is yet to be established. A simulation available online<sup>¶</sup> shows that optical devices may soon reach the continuous tuning speeds necessary for a visual laboratory test of the theory, and also that radio frequency tests and applications should at most require modified RF processing and would be well in reach of existing technology. The implications for astronomical spectroscopy and cosmology (Section 8) concern subcycle drifts of scale even over lengthy photon integrations, and should therefore hold regardless of immediate success or failure in terrestrial validation.

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<sup>§</sup>With  $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , the linear Hubble’s law gives  $H_0 \times 384,400 \text{ km} = 2.87 \text{ cm y}^{-1}$ . This is relativistically naïve, because the relativistic model anticipates no expansion on short scales [28, p719], and the gravitational deceleration originally expected should have reduced the expansion to virtually vanish even at 1 AU [29], but is correct for the present theory in which the recessions are merely apparent. The excess fits a known mismatch factor of 5 with laboratory frictional coefficients [30], currently attributed to previously missing model detail [31, 32]. Since tidal dissipation concerns energy, the corrected apparent recession would be  $3.82 \times \sqrt{5}/(1 + \sqrt{5}) \approx 2.64$ , leaving a margin of  $1.18 \text{ cm y}^{-1}$ , for the real recession, closer to historical recession rates indicated by paleontological data that would not have been affected by phase acceleration, of about  $1.27 \text{ cm y}^{-1}$  from  $-2.5 \text{ Gy}$  to  $-650 \text{ My}$  [33, 34]. This mismatch too is currently attributed to the lack of small seas in the past. Correcting for time dilation due to present clock drift brings the values even closer in line.

<sup>¶</sup>As a Java applet at <http://www.inspiredresearch.com>. This was used for the test results presented in Section 7.

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